ENGINEERING FOR ARCHITECTS

DE WITT CLINTON POND

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Preface

ARCHITECTS often encounter problems in engineering that can be solved with the aid of simple mathematics and a handbook, published by a steel manufacturing company. It is the case, however, that for a certain problem the method of attack is unknown, and the architect is forced to go to an engineer or else risk failure of his structure. In some cases unnecessary cost is incurred through lack of knowledge of the supporting strength of structural members, and the need of such knowledge is felt.

It is to furnish such information that this book has been written. The author does not pretend to introduce any new methods of calculation, nor to give the only ones that may be used. He is simply placing at the disposal of architects such information as will make possible the design of floor beams, girders, column sections, grillage beams, and simple roof trusses. There are, of course, shorter methods that experienced engineers employ; there are entirely different ways in which structural members may be designed; but in case nothing whatever is known of design, it is the hope of the author that this book will give such information as will make the solving of simple engineering problems possible.

D. C. POND

COLUMBIA UNIVERSITY, 1915.

Contents

Engineering for Architects

CHAPTER I

Failure of beams. Definition of "Moment." Formula for the maximum bending moment caused by a uniform load. The use of handbooks.

THE object of this book is to explain simply, and plainly, a few secrets of engineering, so that an architect can decide for himself the size of beams, girders, and column sections necessary for construction. The solutions of these secrets are very simple, and no knowledge of mathematics beyond simple algebra is necessary. If the architect or draftsman will follow these discussions carefully, and bear in mind that all of this work is by no means difficult, he will find there will be no secrets of engineering that he cannot solve for himself. The architect is cautioned to keep from making a mystery of an essentially plain subject.

A handbook, such as published by the Carnegie or Cambria Steel Companies, should be obtained. The possession of such a book is absolutely necessary. A slide rule may be used, but for the simple calculations that an architect may find necessary, a slide rule is hardly essential.

For the first problem we can take a beam, properly braced with tie rods, which carries a brick wall (Fig. 1). The span, usually denoted by the letter "l," will be taken as twenty feet, the wall to be eight inches thick and fifteen feet high. The question is to find the size of an I-beam which will carry such a load. It will be assumed that the wall is fresh laid — the mortar "green" — and that there is no "arch action" caused by the bonding of the brick.

The wall is considered as distributing its weight uniformly over the entire beam, and the beam is said to be "uniformly loaded." In such a case, one half the load is carried by each support. The wall is twenty feet long, fifteen feet high, and two-thirds of a foot thick and the number of cubic feet it contains is given by the following simple multiplication, $\frac{2}{3} \times 15 \times 20 = 200$ cubic feet.

Engineers consider brick as weighing one hundred and twenty pounds per cubic foot, although the New York Building Code gives the weight as one hundred and fifteen pounds. The weight of the wall is, therefore, $200 \times 120 = 24{,}000$ pounds.

The weight coming down on each pier is twelve thousand pounds, and each load is known as the left, or right, reaction. The left reaction is usually denoted as R_1, the right reaction as R_2, and the equation that expresses the above relations is $R_1 = R_2 = 12{,}000$ pounds.

In this case there are two things to be considered: first, the stresses set up in the beam by the external loading — the brick wall,

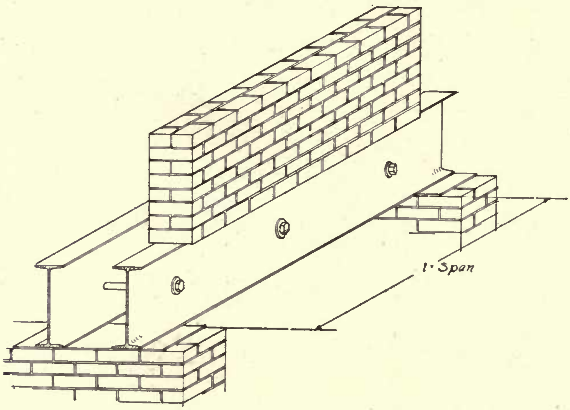


FIGURE 1

— and, second, the ability of the beam to resist these stresses. The architect must keep these two considerations clear and absolutely distinct in his mind.

Taking up the first consideration, engineers generally assume that the external load can make a beam fail in two ways: first, by bending, as shown in Fig. 2 and, second, by shearing, as shown in Fig. 3. Shearing means that type of failure caused by forcing the particles of a beam to slide by each other. When a hole is punched in a steel plate the metal is "sheared" out. Bending sets up stresses of compression and tension which will be considered in the following chapters.

It is obvious in this case that if the beam fails by bending, it will fail in the middle (point c, Fig. 2), ten feet from either support. The tendency to produce bending in the beam, at this point, is found in the following manner.

When bending is considered, the term "moment" is always used, and, at this point, the first definitions must be agreed upon.

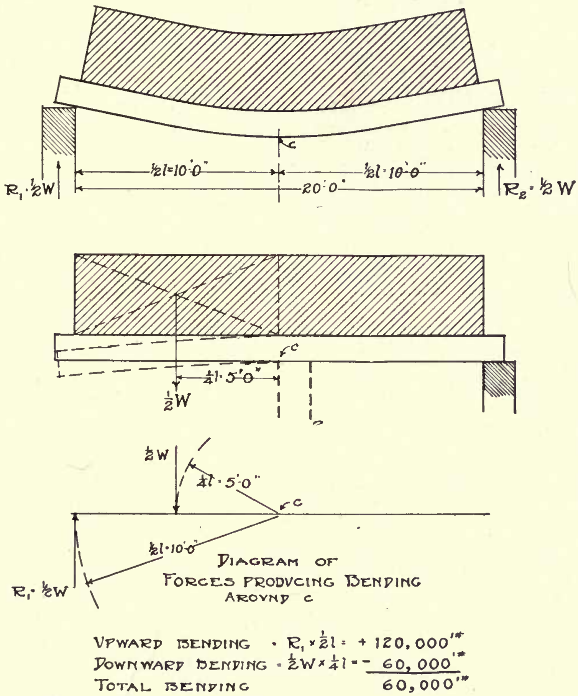


FIGURE 2

A moment is the *tendency to produce rotation* about a point. The point is known as the "center of moments" and the moment itself is equal to the product of the force, tending to produce rotation, multiplied by its *perpendicular* distance from the center of moments, known as the lever arm. In the language of college students, a moment equals "force times distance" or "force times lever arm."

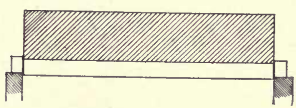


FIGURE 3

A scale is shown in Fig. 4. Either weight W or W' can be taken as the *force*, the point c is the *center of moments*, and the distance d or d' is the perpendicular distance from W or W' to c. If W should drop to the position shown by the dotted lines, d would equal zero and $W \times d = 0$. There would be no tendency to rotate around c and in other words there would be no moment. No force, acting *through* a point, produces a moment.

Since moments are the products of forces and distances they must be measured in units of force and distance, such as "foot-pounds" or "inch-pounds." If a force of five pounds acts at a distance of two feet from a point, the moment will be ten foot-pounds or one hundred and twenty inch-pounds.

No bending is produced unless a moment exists, hence moments are often referred to as "bending moments" and are denoted by the etter M. The force is usually denoted by the letter W and the perpendicular distance, or lever arm, by small l. So $M = Wl$.

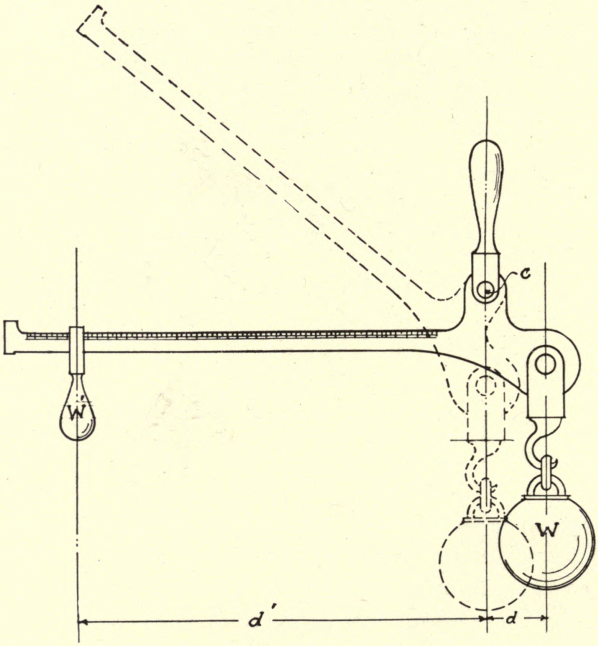


FIGURE 4

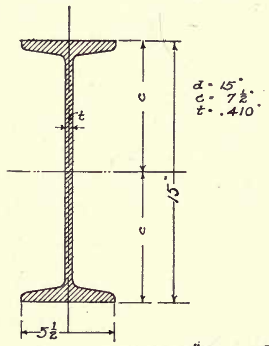
If the beam in Fig. 1 is to fail by bending there must be a moment to cause this bending. The moment must act around the center of the beam (c). Now the left reaction (R_1), may be considered as an *upward* force and the moment caused by R_1 equals the force, 12,000 pounds, multiplied by the distance, 10 feet, or 12,000 pounds $\times$ 10 feet = 120,000 foot-pounds.

If R_1 were taken from its position at the end of the beam and placed, as shown by dotted lines in Fig. 2a, just to the right of c, it is obvious that the section of the brick wall at the left of c would

tend to bend the beam *downward.* The moment tending to produce this bending, of course, must be the product of a force multiplied by a distance. The force in this case is equal to the weight of one half of the wall or $\frac{1}{2}W = 12{,}000$ pounds. The lever arm is the distance from c to the center of gravity of this half of the wall or, in this case, five feet, which is equal to $\frac{1}{4}l$. The moment equals 12,000 pounds multiplied by 5 feet or 60,000 foot-pounds. This downward moment exists around c in all cases even with R_1 in its proper position.

With the reaction acting *upward* and one half the load acting *downward,* the total moment around c must be $(R_1 \times 10) - (\frac{1}{2}W \times 5)$ or $120{,}000 - 60{,}000 = 60{,}000$ foot-pounds or $60{,}000 \times 12 = 720{,}000$ inch-pounds. It is obvious that the bending moment at the right of c is equal to that at the left or $(R_1 \times 10) - (\frac{1}{2}W \times 5) = (R_2 \times 10) - (\frac{1}{2}W \times 5) = M$.

If this were not so, rotation would take place around the center point c which, in the case of a beam, never happens.



SECTION OF A 15"-I-42#

FIGURE 5

The above discussion is given to show a method of finding the maximum bending moment caused by a uniform load on a beam, but, as a rule, this method is too long, and a formula can be used that will give the same results with much less work. The method of deriving the formula is much the same as used in the above discussion. $R_1 = \frac{1}{2}W$, and the moment of R_1 around c equals $R_1 \times \frac{1}{2}l$ or $\frac{1}{2}W \times \frac{1}{2}l = \frac{1}{4}Wl$. The downward moment equals $\frac{1}{2}W \times \frac{1}{4}l$ or $\frac{1}{8}\ Wl$. So the total bending moment around c equals $\frac{1}{4}Wl - \frac{1}{8}Wl$ or $\frac{2}{8}Wl - \frac{1}{8}Wl = \frac{1}{8}Wl$.

The formula, $M = \frac{1}{8}Wl$, is the one always used to find the maximum bending moment in a beam carrying a uniform load. It is, perhaps, the most useful formula employed by engineers and its application in the above case would be as follows:

$W = 24{,}000$ pounds, $l = 20'\text{-}0'' = 240''$, $\frac{1}{8}Wl = \frac{1}{8} \times 24{,}000 \times 240 = 720{,}000$ inch-pounds.

If the beam is to fail by bending it will fail because of this bending moment of 720,000 inch-pounds and if we wish to select one that will carry the brick wall a beam must be selected that will resist this moment. This is the consideration spoken of in the first

part of this article, namely, the ability of the beam to resist the stresses set up by the external load.

The bending moment that a beam will withstand is given by the formula, $M = S\ I/c$.

M is the bending moment, S is the *safe working stress* of the material of which the beam is made. In the case of steel, S is always

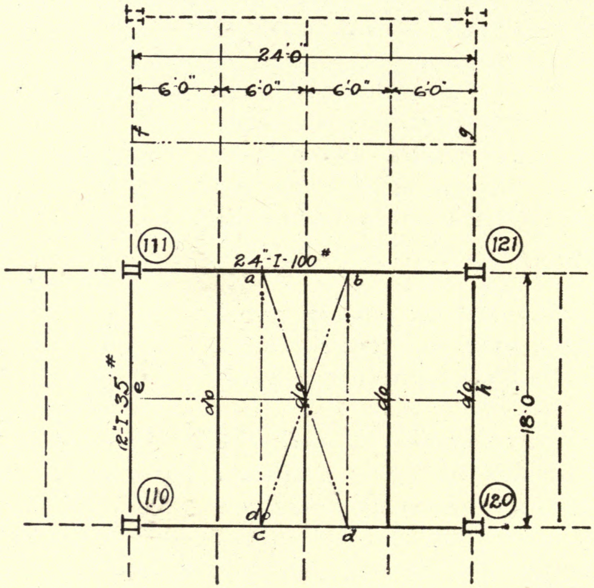


FIGURE 6

taken as sixteen thousand pounds per square inch and in the case of wood from eight to twelve hundred pounds per square inch is all that is allowed. These safe working stresses are given in the building code. I is the symbol always used to denote the "Moment of Inertia" which will be explained later. The letter c is the distance from the most remote fiber to the center (center of gravity) of the beam.

Open the Cambria Steel Company's handbook, 1912 edition, to page 158, or an old Carnegie handbook to page 97, or the "Pocket Companion," published by the Carnegie Steel Company, 1913 edition, to page 142. In the second column from the left, under the heading "Depth of Beam," a fifteen-inch I-beam is found that weighs 42 pounds (Cambria) or 36 pounds (Carnegie, 1913). The Cambria beam will be considered first.

The area of the cross section is 12.48 square inches. The thickness of the web, "t," is 41/100 of an inch. The flange is $5\frac{1}{2}$ inches wide and the moment of inertia or I is 441.8. Obviously, as the depth of the beam is fifteen inches, the distance c (Fig. 5) is $\frac{1}{2} \times 15 = 7.5$. So $I/c = \frac{441.8}{7.5} = 58.9$.

I/c is known as the "section modulus" of the beam, and this is given in column 8, so for any steel beam it is unnecessary for the architect or engineer to go through this simple form of division.

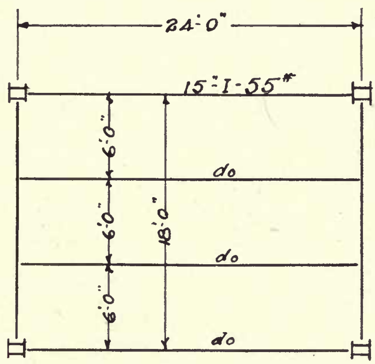


FIGURE 6a

In the formula $M = S\ I/c$, for any standard section of I-beam, channel, or angle, we have every factor known but the bending moment, M. For this particular fifteen-inch, forty-two pound I-beam the moment can be determined by simply substituting the formula as follows: $M = 16{,}000 \times 58.9 = 940{,}000$ inch-pounds, approximately, and this is the bending moment that the beam will withstand. As a rule, M is known and I/c, the section modulus, is the unknown factor. Such is the case with the simple beam carrying the brick wall. M, in this case, is 720,000 inch-pounds. To determine the size of the I-beam required, the formula would be written as follows: $M = 720{,}000$ inch-pounds $= 16{,}000 \times I/c$, or $I/c = 720{,}000/16{,}000$. $I/c = 45$.

Looking down column 8 in the handbook 58.9 is the next value for I/c above 45, so a fifteen inch, forty-two pound I-beam is required to carry the wall.

If we consider the Carnegie handbook, the fifteen-inch I-beam, weighing thirty-six pounds per foot, will have a section area of 10.63 square inches, a flange width of $5\frac{1}{2}$ inches, with a web thickness of 289/1,000 of an inch. The I, or moment of inertia, of this beam is 405.1, and as $c = \frac{1}{2} \times d = 7.5$, $I/c = \frac{405.1}{7.5} = 54$, which is found in the column marked "S" in the Carnegie book. "S," in the handbook, denotes the section modulus or I/c. So a fifteen inch, thirty-six pound I-beam would withstand a maximum bending moment of $M = 16{,}000 \times 54 = 864{,}000$ inch-pounds.

Following the calculations given for the Cambria beam it is easily seen that this beam will carry the brick wall.

The second method of failure is due to shearing and in this case the beam might fail in this manner rather than by bending, as a brick wall is practically rigid after the mortar is set.

The shear is always greatest at the supports and is equal to the reactions. Looking at Fig. 3 it might be imagined that the wall acts like a punch — tending to punch the beam between the two supports, and the force exerted at each pier is equal to each reaction.

Therefore the maximum shear in the beam equals 12,000 pounds.

Steel will safely stand a tensile stress or pull of 16,000 pounds per square inch, but its shearing value is only 9,000 pounds per square inch, as allowed by the New York Building Code, Section 139.

The cross-section area of the Cambria beam, as given in the handbook, is 12.48 square inches. If the shearing strength of the steel is 9,000 pounds per square inch and there are 12.48 square inches, naturally the shearing value of the cross section is $12.48 \times 9{,}000 = 109{,}320$ pounds. The beam is undoubtedly safe.

It is a significant fact that although moments are measured in foot-pounds or inch-pounds, foot-tons or inch-tons, as the case may be, shearing values are measured in pounds or tons only. If the architect finds that he is trying to measure moments in terms of weights only, he had better stop and consider that it would be as impossible to do this as to measure a mile with a quart measure. The unit does not fit the condition.

The maximum shear is always found at the reactions and when the reactions are unequal the greatest shear is found at the greatest reaction.

As a rule beams are seldom figured to withstand shear as long beams are more apt to fail by bending. However, in the case of heavy loads, carried over short spans, failure by shearing must always be considered. The bending moment caused by a heavy load over a short span will be small as the maximum moment equals $\frac{1}{8}Wl$ and l in this case is small. If the beam is designed only for the purpose of resisting this M, it might easily be too light to resist the shear.

In the foregoing problem, a single case is given where a uniformly distributed load is considered. Another, and probably the most frequent case, is found where the beam is one of a series of "filling-in beams," as shown in the section of a steel framing plan (Fig. 6). A

girder is "framed" between columns 120 and 110, and another between columns 121 and 111. Filling-in beams, spaced six feet on centers, are framed between the girders. The floor area carried by each beam is shown enclosed in the space *abcd*, and is equal to 6′-0″ × 18′-0″ = 108 square feet. Assuming a floor load of 200 pounds per square foot, then the total load on each beam is 108 × 200 = 21,600 pounds = W.

The span of the beam is 18′-0″ or 216″ and so the bending moment equals $\frac{21{,}600 \times 216}{8} = 583{,}200$ inch-pounds,

$$M = 583{,}200 = S\,I/c = 16{,}000 \times I/c. \quad I/c = \frac{583{,}200}{16{,}000} = 36.4.$$

In the handbook a twelve inch, thirty-five pound I-beam will be found having a section modulus of 38.0 which will be satisfactory for this condition.

The architect can make problems for himself, such as assuming a floor load of 175 pounds or a span of 20 feet in place of 18 feet. If the beams and girders were framed in opposite directions, as shown in Fig. 6a, the beam would have a span of 24 feet and therefore a 15″ I-beam, weighing 55 pounds per foot, would be necessary.

If, in Fig. 6, we *assume* that the girders carry a section of the floor area enclosed by the lines *ef*, *fg*, *gh*, and *he*, the floor area will be 18 feet by 24 feet, or 432 square feet. 432 × 200 = 86,400 pounds = W.

$$\tfrac{1}{8}Wl = \tfrac{1}{8} \times 86{,}400 \times 24 \times 12 = 3{,}110{,}400 \text{ inch-pounds.}$$

$$M = 3{,}110{,}400 = 16{,}000 \times I/c, \text{ or}$$

$$I/c = \frac{3{,}110{,}400}{16{,}000} = 194.4.$$

Therefore, the girders would have to be 24″ I-beams, weighing 100 pounds per foot.

The method of figuring floor loads as well as a little more theoretical consideration of the principles involved in the above discussion will be taken up in the next chapters. For the present it may be well for the architect to "check up" the sizes of beams shown on any steel framing plans that he may have.

CHAPTER II

Wood beams. Moment of Inertia. Safe loads. Floor Construction.

THE two formulas given in the first chapter, $M = S\ I/c$ and $M = \frac{1}{8}Wl$, are the two most common formulas used in engineering work. The "I," used in the first, stands for the moment of inertia of the cross section of the beam. To give a definition of this term would be useless as it would only cause confusion. For any steel beam the moment of inertia is given in the handbooks published by the steel companies. If, however, the architect should want to know the size of a wooden beam, strong enough to carry a given load, he must find this factor for himself. This is by no means a difficult task.

The cross section of all wooden beams is rectangular, and, for this kind of a section, the formula that gives the moment of inertia is $I = 1/12\ bd^3$, in which b denotes the breadth of the beam, and d denotes the depth. So, for a $2'' \times 10''$ wooden joist, the formula would be written, $I = 1/12 \times 2 \times 10 \times 10 \times 10$, and I, for this beam, would be 166.6. This is all that is necessary for an architect to know about moments of inertia.

I/c, the section modulus, is found by using the formula $I/c = 1/6\ bd^2$. If the architect bears in mind that c equals one half of d, and that $I = 1/12\ bd^3$, he can easily derive this formula for himself. The section modulus of the joist can be determined in the following manner: $I/c = I/6 \times 2 \times 10 \times 10 = 33.3$. This checks with the fact that 166.6 divided by 5 equals 33.3.

To find the safe uniform load that the joist will carry over a span of ten feet the formula $M = \frac{1}{8}Wl$ is used:

$$\frac{1}{8} \times W \times 10 \times 12 = S\ I/c = 1{,}200 \times 33.3.$$
$$\frac{1}{8} \times W = 1{,}200 \times 33.3/120.$$
$$W = 333.3 \times 8 = 2{,}666 \text{ pounds.}$$

In the steel handbooks, under the heading of "Safe Loads in Pounds for Wooden Beams" or simply "Wooden Beams," loads are given for joists, one inch thick, for various spans and depths. On page 352 in the Cambria Steel Company's handbook, the safe

load for 1″ × 10″ wood beam spanning ten feet, is given as 1,333 pounds. A 2″ joist would be twice as strong and would carry a load of 2,666 pounds, which checks with the answer given above. It is suggested that the architect figure the safe load for a 3″ × 12″ beam, having a safe working stress of 1,000 pounds per square inch, and a span of 15′-0″. The load should work out to be 3 × 1,067 or 3,201 as given on page 350 of the Cambria book.

The calculation for the above problem is very simple and should take not more than three or four minutes.

Of course the safe loads for wooden beams are given and it is really unnecessary for the architect to determine them, but the calculations serve to make the terms, "moment of inertia" and "section modulus," familiar ones. The importance of this familiarity cannot be too strongly emphasized as the process of finding the sizes of beams and girders should be almost automatic.

The real problem that any designer has to solve is to determine the *kind* and the *magnitude* of the loads. Once these questions are settled, a method of finding the size of beam to carry the loads should take but little thought.

There are two kinds of loads — uniform and concentrated. So far we have only dealt with the first, but later we will take up the problem of finding beams suitable for carrying concentrated loads.

The processes of finding magnitudes of loads will be taken up at once.

There are two *types* of loads, dead loads and live loads. The dead loads include the weight of walls, and weight of the floor construction. These weights are usually known as floor loads and wall loads. In Fig. 7 the wall load is carried on beams known as spandrel beams, and the floor load is carried on filling-in beams. In some cases, however, the spandrel beams have to carry some of the floor load as well as the wall load.

The floor construction, in this case, is made of a six-inch terra cotta arch, between the beams, on which cinder fill is carried. The cinders fill in over the haunches of the arch and are levelled off at the tops of the *I*-beams. On the cinders wooden sleepers are placed, between which, cement mortar is laid. The wooden flooring is nailed to the sleepers and a hung ceiling is suspended from the lower flanges of the *I*-beams.

Under the heading "Arches" in the handbooks the weight of terra cotta arches is found. It will be found that a six-inch arch is

given as weighing twenty-seven pounds per square foot of floor area. Also, under the heading of "Weights," the weight per cubic foot is given of cinders, cement, and wood.

The beams will be taken as being twelve inches deep, spaced six feet on centers (Fig. 9). The rise of the arch will be an inch

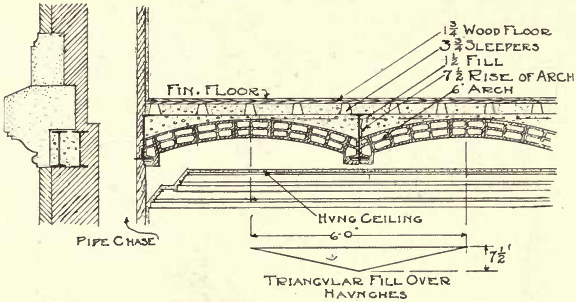


FIGURE 7

and a quarter per foot of span as required by the New York Building Code. For a span of six feet the rise (Fig. 7) will be six times one and one-quarter or seven and one-half inches. The top of the arch will be $6'' + 7.5'' = 13.5''$ above the bottom flange of the *I*-beam or $1.5''$ below the top flange, in case a $15''$ I-beam is used.

There will be a triangular fill over the haunches of the arch and a "straight" fill of an inch and a half over the tops of the arches to the tops of the beams. All of this fill will be made of cinders weighing 45 pounds per cubic foot. As is shown in Fig. 8, a layer of cinders, one inch deep and a foot square, will weigh one-twelfth of 45 pounds, or, approximately, 4 pounds.

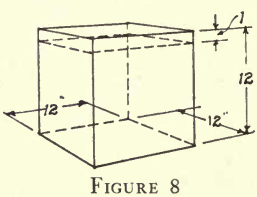


FIGURE 8
A Layer 1″ thick and a foot square weighs one twelfth as much as a cubic foot.

The filling over the haunches is roughly in the shape of a triangle, having a base of six feet and an altitude of seven and a half inches. This triangle is equivalent to a straight fill of three and three-quarters inches deep. Add to this the inch and one-half at the top of the arch and the sum will be five and one-quarter inches.

On top of the cinders the sleepers are placed and the space between the sleepers is filled in with cinder concrete. Engineers consider this portion of the floor construction as weighing the same as

the cinder fill. To the 5.25″ add 3.75″ and the sum is 9.00″. This is the average depth of the fill between the arch and the rough flooring, and as each inch weighs 4 pounds the total weight is 36 pounds.

The average weight of a cubic foot of wood can be taken as thirty pounds. A layer, one inch thick and a foot square, will weigh one-twelfth of thirty or two and one-half pounds. There is an approximate thickness of wood flooring of two inches, which will weigh five pounds per square foot of floor area.

There still remains the weights of the ceiling and the steel beams and girders to be considered. A hung ceiling is always supposed to weigh ten pounds per square foot so this item is easily disposed of, but to understand the method of figuring the weight of the steel as a part of the floor load, it is necessary to study the framing plan (Fig. 9). The portion of the floot enclosed in the area *a b c d* is the size of a floor panel, 18′-0″ × 24′-0″, and in it are found portions of beams and girders, such as those that make up the average panel. There is a part of a twenty-four inch *I*-beam weighing one hundred pounds per foot. The length of the beam found in area *a b c d* is 24 feet, so the total weight of the girder will be 24 × 100 = 2,400 pounds. There are four filling-in beams, each of which is 18′-0″ long and weighs forty pounds per foot. The total weight of the beam is then 4 × 18 × 40 = 2,880 pounds. Add this to 2,400 pounds and the steel in the panel will weigh 5,280 pounds. There are 432 square feet in the panel, so the weight per square foot of floor area of the steel will be 5,280 ÷ 432 = 12.2 pounds. To be safe, and to give an approximate figure for the steel, this weight will be taken as 13 pounds.

To find the weight of a square foot of floor construction add all the weights given above.

10	pounds	=	weight of ceiling.
27	"	=	" " terra cotta arch.
36	"	=	" " cinder fill.
5	"	=	" " wood floor.
13	"	=	" " steel
91	"	=	Total weight of floor.

The ninety-one pounds is the total dead load per square foot but there is still a live load to be considered. The live load is always given in the building codes of the different cities and all that the

architect has to do is to find what live load per square foot is required for the particular building he is designing.

The New York Building Code, in section 130, gives the live load on floors of dwellings and apartment houses as not less than sixty pounds. If a building is to be used as an office building, the live load is figured as seventy-five pounds on all floors above the first, and on that the engineer must figure for a live load of one hundred

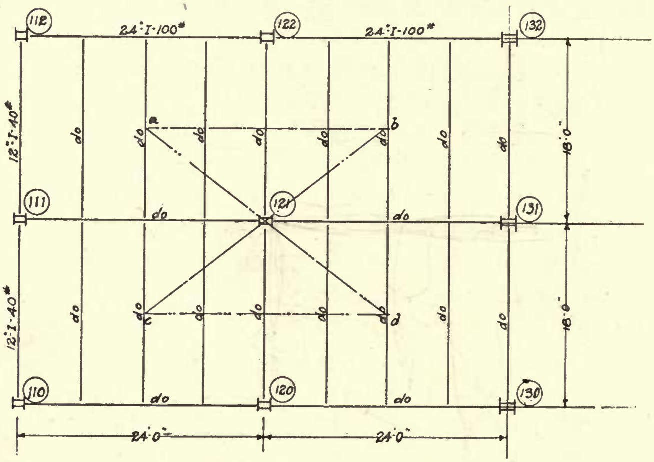


FIGURE 9
To determine the weight of steel for the Dead Load these sizes are assumed.

and fifty pounds. Floors of schools must be strong enough to carry seventy-five pounds per square foot. A live load of ninety pounds is allowed on floors of places of public assembly, and store floors must carry one hundred and twenty pounds per square foot.

In case the floor construction, given above, is to be used in a department store, the live load upon it will be 120 pounds, so the total weight per square foot of floor area will be $91 + 120 = 211$ pounds. As an easy figure to deal with, this total load can be taken as 210 pounds.

In the floor plan (Fig. 9) the weight carried by each twelve-inch beam will be $6 \times 18 \times 210 = 22{,}680$ pounds. The bending moment will be $\frac{1}{8}Wl = \frac{1}{8} \times 22{,}680 \times 18 \times 12 = 612{,}360$ inch-pounds. Dividing this by the safe working stress of steel — 16,000 pounds per square inch — we will get a value for I/c of $612{,}360 \div 16{,}000 = 38.2$. The section modulus, or I/c, as given in the steel companies' handbooks, for a 12″-1-40lb will be a little larger than this.

The type of floor arch that we have been investigating is only one of many that are used in floor construction. Where it is desirable to plaster directly on the soffit — the under side — of the arch, flat terra cotta arches are used. The New York Building Code, in section 106, contains the statement that the "depth shall not be less than one and three-quarter inches for each foot of span, not including any portion of the depth of the tile projecting below the under side of the beam, if the soffit of the beam is straight." This means that for a span of six feet the arch must be ten inches and a half deep above the bottom flange of the beams. If an absolutely flat ceiling is desired, to this ten and one-half inches the depth of the fireproofing on the lower flange of the beams must be added. This fireproofing can be considered as being an inch and one-half thick so the total depth of the arch will be $10.5 + 1.5 = 12''$.

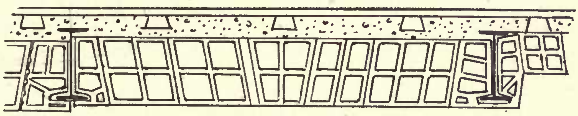


FIGURE 10
Flat Terra Cotta Floor Arch.

In the terra cotta manufacturing companies' handbooks the depth of arch required for the above condition is given as ten inches. Although the twelve-inch depth is excessive it must be used in New York. The weight of a twelve-inch arch can be taken as 38 pounds per square foot of floor area. If the arch is sprung between fifteen-inch beams (Fig. 10), the distance between the tile and the wooden flooring is fourteen inches, approximately. As each inch of fill weighs four pounds, the weight will be $4 \times 14 = 56$ pounds. The flooring and steel will weigh the same as in the first case, so the total weight will be:

38	pounds	=	weight of arch.
56	"	=	" " fill.
5	"	=	" " floor.
13	"	=	" " steel.
112	"	=	" " a square foot of floor.
120	"	=	" " live load.
232	"	=	Total.

Taking the load as 230 pounds per square foot of floor area, the section modulus of 41.8 is obtained. A fifteen inch, forty-two

pound I-beam will be required to take this load. As there is only a difference of two pounds between this and the twelve-inch beam, it will be practically as cheap to use the deeper section.

It will be noticed that in both Fig. 7 and Fig. 10 the arches are sprung between fifteen-inch beams, although twelve-inch filling-in beams are often shown in the plans. The reason for the use of the

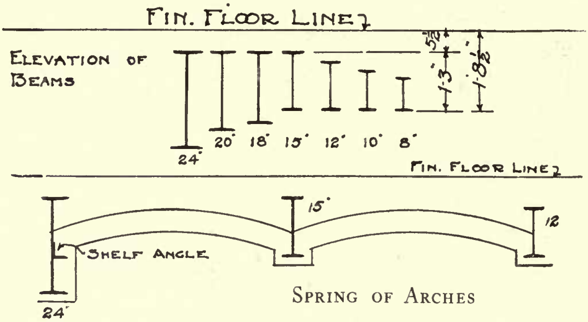


FIGURE 11

deeper beams is as follows: on all framing plans the relative heights of steel beams are shown in the same manner as given in Fig. 11. All beams, having a greater depth than fifteen inches, are shown "flush top" with the fifteen inch I-beams. This means that the top flanges of these beams are all on the same level, usually from five to six inches below the finished floor line. All beams having a depth of less than fifteen inches are shown "flush bottom" with the fifteen-inch beams. As nearly all arches spring from 15″, 12″, or 8″ filling-in beams, this arrangement makes it possible for the bottom of the arches to be on the same level. If the difference of level between the bottom flanges of the beams is more than three inches, it is impossible to spring an arch between them, and a

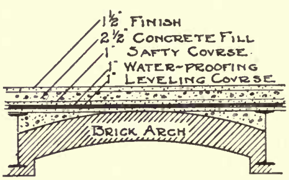


FIGURE 12

This is very heavy construction. Side Walk Loads are seldom figured to be more than 400 pounds per square foot.

"shelf angle" must be used as seen in Fig. 11. In other words, an arch can be sprung between a 15″ beam and an 18″ beam if the upper flanges of these beams are "flush" but a shelf angle must be used if it is desired to spring the arch between a 15″ beam and a 20″ or a 24″ beam. The fifteen-inch beam is considered as a standard.

The method of finding the weight of a square foot of floor construction, as given above, is the one used in all cases where floor loads are considered. Sidewalk loads and roof loads differ from floor loads as brick arches are used and the construction must be waterproof.

In Fig. 12 a typical form of sidewalk construction is shown. When a concrete finish is desired a cinder concrete fill is used. The weight for a square foot of sidewalk construction is obtained as follows:

$3\frac{1}{2}''$ cement at 10lb per inch	=	35lb
$2\frac{1}{2}''$ concrete fill at 7.5lb	=	19"
$1''$ waterproofing	=	5"
$3\frac{3}{4}''$ cinder fill at 4lb	=	15"
$8''$ brick arch at 115lb	=	77"
60lb beam — 6'-0" span	=	10"
Total dead load	=	161"
Live load	=	300"
Total	=	461"

There are many different kinds of sidewalk, roof, and floor construction, and, of course, the architect must know the type that is being used in any building he is designing. If he employs the method given above, he can easily determine the weight of the floor even though the dimensions or the materials may be different.

Wall loads are no harder to determine than floor loads, but, as walls are cut up by windows, the work required to find the points where the greatest load is going to bear upon the wall beam is tedious. If accuracy is required, it is necessary to study every panel of the wall. As the method of determining the beam to carry a wall, where the loading is not uniform, involves the consideration of concentrated loads, wall loads will be taken up in the next chapter.

As problems for practice, it is suggested that the architect determine for himself whether the beams in any framing plan that he may have are of the proper size to carry the loads found by him.

CHAPTER III

Concentrated Loads. Reactions. Points of maximum bending moment, and shear diagrams. Method of finding M, the maximum bending moment. Method of finding the section modulus when M is measured in foot-tons. The design of spandrel beams.

TO determine the size of beams strong enough to carry a series of concentrated loads, it is necessary to study the theory of the subject. Our American ideal of being practical is not to be condemned, but, unless the architect grasps, to a small extent, the theory back of all this engineering work, he is apt to become the slave of the handbook.

In Fig. 13 a diagram, representing a simple beam carrying three concentrated loads, is shown. The beam has a span of twenty feet and the loads are five, four, and two tons respectively located eight,

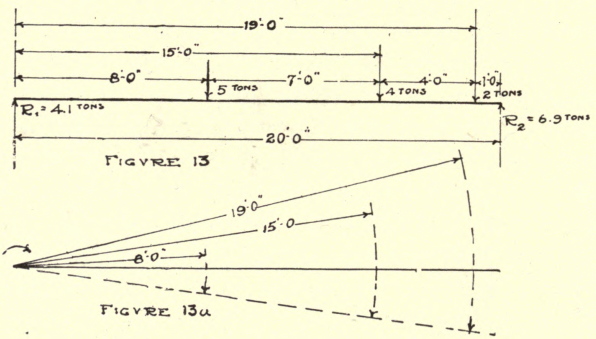


FIGURE 13

fifteen, and nineteen feet from the left support. In Chapter I, where a simple beam was considered, there was a uniformly distributed load and the forces exerted at each support were equal. In this case, however, it is obvious that there will be a different force at R_1 from that at R_2. The loads are placed on the right side of the beam and so, at the first inspection, it might be assumed that the right reaction will be greater than the left. In order to find out if this is true, and, at the same time, to determine the exact amount of loading on each support, the following process is employed.

If R_2 were removed, and the beam were allowed to swing freely around R_1, there would be a tendency to rotate around the left support as shown in Fig. 13a. Remembering the definition, in the first article, that a moment is the tendency to produce rotation around a point, known as the center of moments, it is plainly seen that there would be a moment around R_1, and this will equal the sum of *all* the moments caused by *all* the loads. R_1 becomes the center of moments. The five-ton load will produce a moment of 40 foot-tons around R_1. The four-ton load, having a lever arm of fifteen feet, will produce a moment of 60 foot-tons. A smaller moment of 38 foot-tons will be caused by the two tons located nineteen feet from R_1. As a result the total tendency to rotate around the left support is given as follows:

5 tons	×	8 feet	=	40 foot-tons.	
4 "	×	15 "	=	60 " "	
2 "	×	19 "	=	38 " "	
				138 " "	Total.

Unless an upward moment is used to counteract this downward moment there will be rotation around the left reaction. The only upward force that can possibly produce this moment is the right reaction (R_2). This reaction acts at a distance of twenty feet from R_1, and the upward force that it would have to exert on the beam to produce equilibrium is given by the following equation: $R_2 \times 20$ feet $= 138$ foot-tons, or $R_2 = 138 \div 20 = 6.9$ tons.

To obtain the left reaction, the same method can be employed, the only difference being that R_2 will be taken as the center of moments. In this case the results are:

2 tons	×	1 foot	=	2 foot-tons.	
4 "	×	5 feet	=	20 " "	
5 "	×	12 "	=	60 " "	
11 tons				82 " "	Total.

$82 \div 20 = 4.1$ tons as the load on R_1.

As a rule the left reaction is obtained in a much simpler manner. The sum of the reactions must equal the sum of all the downward loads, otherwise there would be a tendency to push the beam either up or down. If the right reaction is obtained, the left one is determined by simply subtracting R_2 from the total load. By arranging the computation as shown above, the total load is given by plain

addition. From the 11 tons subtract 6.9 tons, and the remainder is 4.1 tons. The second computation is unnecessary except as a check.

Take a second problem, the diagram for which is shown in Fig. 14. Here we have a uniform load as well as two concentrated loads. There need be no hesitation about attacking the new condition, as a uniform load is treated in exactly the same manner as a concentrated one. First obtain the *total* uniform load. Then find the distance from the center (center of gravity) of the load to the point taken as the center of moments. The moment around this center is the product of the total load multiplied by the distance. In other words, the uniform load acts exactly like a concentrated load which has been placed at the center of gravity of the distributed weight. The load per foot is usually denoted by small w, and the total weight by large W. If l is the span, and w the weight per foot, then $W = wl$. If the architect always uses the large W as a basis for his calculation, he will avoid many disturbing complications that follow the use of the smaller letter. In the diagram, shown in Fig. 14, R_2 is obtained by the following calculation:

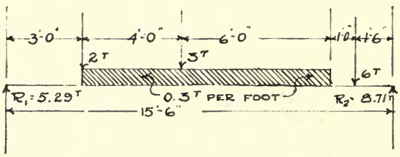


FIGURE 14

$$
\begin{array}{rrcrcl}
 & 2 \text{ tons} \times & 3 \text{ feet} & = & 6 & \text{foot-tons.} \\
 & 3 \quad \text{"} \times & 7 \quad \text{"} & = & 21 & \text{"} \quad \text{"} \\
0.3 \text{ tons} \times 10 = & 3 \quad \text{"} \times & 8 \quad \text{"} & = & 24 & \text{"} \quad \text{"} \\
 & \underline{6} \quad \text{"} \times & 14 \quad \text{"} & = & \underline{84} & \text{"} \quad \text{"} \\
 & 14 \text{ tons} & & & 135 & \text{"} \quad \text{"}
\end{array}
$$

$$135 \div 15.5 = 8.71 \text{ tons} = R_2.$$
$$14 - 8.71 = 5.29 \text{ tons} = R_1.$$

The architect can check the calculations by taking moments around R_2. It is worth noting that if R_1 is taken as the center of moments, R_2 is obtained. *One of the most frequent mistakes of beginners is to give the value of the reaction obtained by the calculations to the reaction used as the center of moments.*

The fact that it is always easy to check results makes it possible for the architect to originate his own problems and be absolutely sure that his answers are correct. He can take spans of any conven-

ient length — twelve, sixteen, or twenty feet — and can assume any kind of loading. If the reactions are found in the proper manner the results will always check.

In case we have a simple beam, unsymmetrically loaded, the first calculations are made to find the reactions. The second process is to find the *maximum bending moment.* When there are concentrated loads, which may be placed in any position whatever, the point where the maximum bending moment is going to occur is unknown.

It is obvious, that when there is a single concentrated load, as shown in Fig. 15, the maximum bending is going to take place directly under the load. Take the point c, at the load, as the center of moments, and determine the tendency to produce bending around this point. The right reaction is 8 tons — check this — and the distance from c is 5 feet. So the bending around c is caused by a moment of 8 tons × 5 feet = 40 foot-tons. The left reaction $(R)_1$ causes a bending moment of 4 tons × 10 feet = 40 foot-tons, which equals that produced by R_2. This is correct, for, if one moment were greater than the other, rotation would take place around c and the beam would not be in a condition of equilibrium.

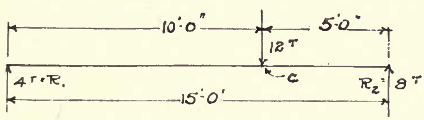


FIGURE 15

The first impulse of a beginner is to use the 12-ton load to find the bending at c. It seems plain that this load causes bending, and, as it creates the loads on the reactions, it does. But the 12-ton load acts *through* the point c. There is no lever arm, and, therefore, it produces no direct bending.

The fact that the maximum bending occurs at the point c is so plain that it needs no further explanation. When, however, there are several loads as in Figs. 13 and 14, it requires some calculation to find the exact spot where the greatest moment is going to occur.

The next statement must be taken by the architect without proof. That is: the maximum bending occurs where zero shear exists. This means that where there is no tendency for the beam to fail by shearing it is most apt to fail by bending. To prove this it is necessary to resort to calculus, but all that the architect need realize is, that to find the point of greatest bending, it is necessary to find the point of no shear.

Fig. 3, in Chapter I, shows a beam, uniformly loaded, failing by shearing at the supports. In any case the greatest shear is found

at the reactions and in the case of the uniformly distributed load there can be no tendency to shear at the center of the beam, as the upward reactions ($R_1 = R_2 = \frac{1}{2}W$) would be counteracted by the downward weight of one-half of the wall at the right or left of

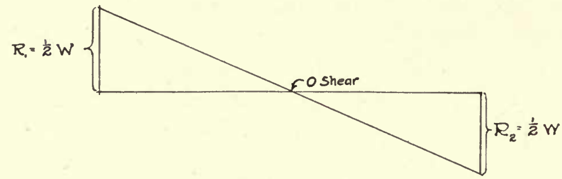


FIGURE 16

the center. At this point the downward force becomes equal to the upward force and the shear becomes zero. The diagram that expresses this is shown in Fig. 16. Lay off a distance, upward, at R_1 equal to the left reaction, and lay off at R_2 a distance downward, equal to the right reaction. At the center there is no shear. Connect the points with a straight line and the shear diagram is drawn. The shear at the left support equals R_1 and steadily decreases as the distance increases away from the reactions, until at the center the shear is zero. From there on the shear increases until it reaches the right support where it equals R_2. The line representing the shear caused by a uniform load is always a sloping line and the total drop is equal to the total load.

Concentrated loads have a different diagram. In Fig. 14, if a weak spot should occur at any point between the left support and

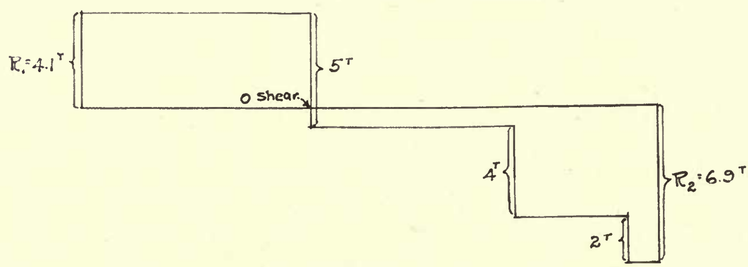


FIGURE 17

the first load, the upward reaction (R_1) would shear off the beam at that point. The shear will remain the same for all points between the reaction and the first load. This is shown in the shear diagram (Fig. 17). At the point where the first load is located, the shear caused by R_1 will be offset by the downward load of five tons. This

will cause a minus shear of 4.1 – 5.0 = – 0.9 tons shear. There would be no change in shear until the next load is reached where the minus shear of — 0.9 – 4.0 = – 4.9 tons will occur. The two-ton load will cause a minus shear of – 4.9 – 2.0 = – 6.9 tons, which will remain the same for all points between the last load and the right reaction. When R_2 is reached the upward force of 6.9 just counteracts the downward (minus) shear.

In Fig. 18 a shear diagram is shown for the loading given in Fig. 14. The only difference between this diagram and that in Fig. 17, is that the uniform load gives a sloping line, instead of a "step" as a concentrated load would. The total drop of the sloping line is equal to the total uniform load. The distance "a" plus the distance "b" should equal 3 tons. The shear diagram gives graphical proof that the sum of the reactions must equal the sum of the loads.

For the beam shown in Fig. 13, the loads will give a maximum bending moment at the point where the five-ton load is located, as

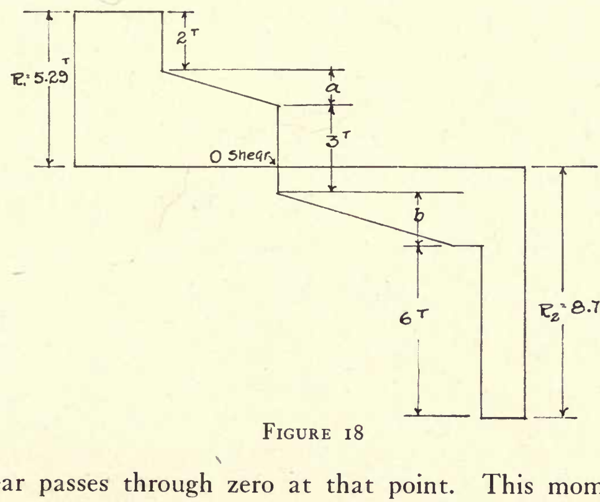


FIGURE 18

the shear passes through zero at that point. This moment must equal 4.1 tons × 8 feet = 32 foot-tons. To prove that this is the maximum, take the moments at the four-ton load. (4.1 × 15) – (5 × 7) = 26.5 foot-tons. The moment under the small load — two tons — will be (4.1 × 19) – (5 × 11) – (4 × 4) = 77.9 – (55 + 16) = 6.9 foot-tons. This result checks with the fact that the moment caused by R_2 around the two-ton load is 6.9 tons × 1 foot = 6.9 foot-tons. Once the maximum bending moment is determined, the

section modulus is found in the following manner: $M = 32$ foot-tons, or, 32×12 inch-tons. S, taken in tons, equals 8 tons. So, $32 \times 12 = 8 \times I/c$. $I/c = 32 \times 12/8 = 48$. A fifteen-inch I-beam, weighing forty-two pounds per foot, will be strong enough to carry this load.

If M is measured in *foot-tons,* to find I/c simply multiply M by $3/2$. To prove this substitute in the formula $M = S \times I/c$. $M \times 12 = 8 \times I/c$ or, $M \times 12 \div 8 = I/c$. So $M \times 3/2 = I/c$. This method can be used *only* when M is measured in foot-tons.

For the beam in Fig. 14 the maximum bending is found at the concentrated load of three tons. M, in this case, equals

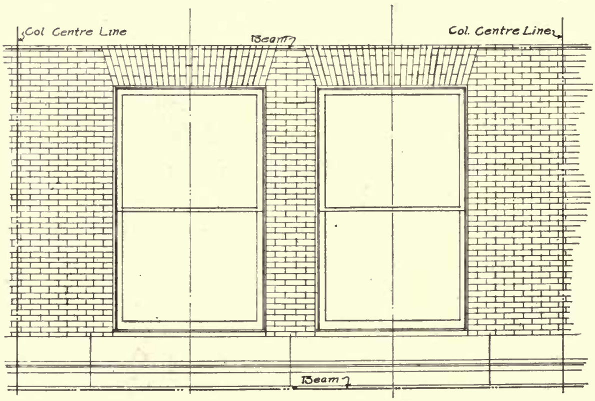


FIGURE 19

$5.29 \times 7 - (2 \times 4) - (0.3 \times 4 \times 2) = 37.03 - 8 - 2.4 = 26.63$ foot-tons. Taking moments at the right as a check, the results are, $8.71 \times 8.5 - (6 \times 7) - (0.3 \times 6 \times 3) = 26.63$ foot-tons. The section modulus is $26.63 \times 3/2 = 39.94$, and a twelve-inch, forty-pound I-beam will be required for these loads.

The processes, given above, can be re-stated briefly, as follows: First, determine the reactions. Second, draw the shear diagram and find the point of no shear. Third, determine the bending moment at this point. Fourth, find the section modulus and the size of beam required.

Now to reduce all the theoretical discussion to practical considerations, consider the wall load, shown in Fig. 19. The wall is one foot thick and is pierced by windows, which are 5′-8″ wide by 9′-0″ high. Below the windows and running the entire length of the

wall is a stone course which is two feet thick. The sill of the windows is two feet above the flange of the beam. The steel columns are 20′-0″ apart and are located so that the loading on the wall beams will be symmetrical. The distance between the upper flanges of the wall beams will be 12′-4″. This gives a wall panel 12′-4″ × 20′-0″. The weight of the brick over the windows is carried down to the beam by the piers and the mullion. These brick weights will be considered as concentrated loads and the stone will be taken as distributing its weight uniformly over the entire beam. The loading of

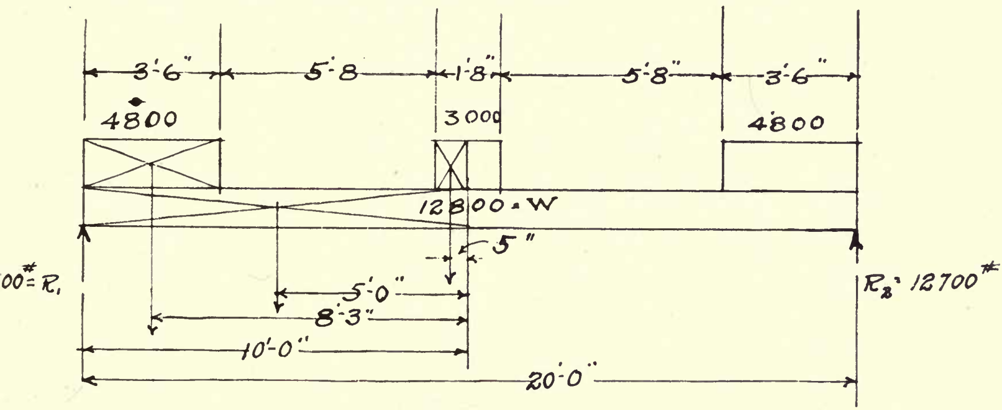


FIGURE 20

the beam is shown in Fig. 20. To get these loads the following process is employed:

The distance from the center line of the columns to the center line of the windows is 6′-4″ and the area of brick included between these lines is (10′-4″ × 6′-4″) minus one-half the window area which is (9′-0″ × 2′-10″). This gives 65.4 square feet minus 25.5 square feet, which equals approximately 40 square feet. As the wall is one foot thick, the weight of this area is 40 × 1 × 120 = 4,800 pounds. This is the load brought down by each pier. The load brought down by the mullion is obtained by finding the weight of the brick area between the center lines of the two windows. (7′-4″ × 10′-4″) − (5′-8″ × 9′-0″) = 24.77 square feet. 24.7 × 1 × 120 = approximately 3,000 pounds. The uniform load of the stone, which is two feet high, two feet thick, and twenty feet long, is 80 × 160 = 12,800 pounds. As the loads are symmetrically distributed, each reaction will be equal to one-half of the total load, or 4,800 + 3,000 + 12,800 + 4,800 = 25,400/2 = 12,700 pounds.

The maximum bending moment will be in the center of the beam and will be equal to the sum of all the moments taken at the left of this point. $12{,}700 \times 120'' - (4{,}800 \times 99'' + 12{,}800/2 \times 60'' + 1{,}500 \times 5'') = 1{,}524{,}000$ inch-pounds $- 866{,}700$ inch-pounds $= 657{,}300$ inch-pounds.

To find the section modulus of the beam, divide the bending moment by the stress per square inch allowed for steel – 16,000 pounds. $657{,}300 \div 16{,}000 = 41$ approximately. A twelve-inch I-beam, weighing forty pounds per foot will be strong enough to carry the loads. From the above calculations, it can be seen that it is much simpler to use foot-tons rather than inch-pounds.

There are all kinds of conditions where concentrated loads occur. When there is framing around elevator shafts, and other cases where beams frame into girders in a manner that gives unsymmetrical loading, it is necessary to use the methods employed in this article. The method of drawing bending moment diagrams and the considerations involved in designing a built-up girder will be taken up in the next chapter. For the present it would be well for the architect to become thoroughly acquainted with shear diagrams and points of maximum bending moments.

CHAPTER IV

"Built up" Girders. Depth of girder. Thickness of web plate. Maximum bending moment. Area of flanges. Length of cover plates and bending moment diagram. Stiffeners.

THE standard beams, rolled by the Steel Companies, are strong enough to be used under ordinary conditions, but when the spans are very great and the loads unusually heavy, these beams cannot be made use of. Special girder beams are now being rolled which withstand much more loading than the ordinary I-beam, but there are many conditions which make it necessary to use riveted girders.

Riveted, or "built up" girders, are made of plates and angles, fabricated in such a manner as to give a section very similar to that of an I-beam. The diagram, shown in Fig. 22, shows the parts of a single web-girder. The flange is made of cover plates and angles and the web is made of a single rolled plate usually from three-eighths of an inch to one inch thick. Girders are made with two and sometimes three webs, but as these are difficult to fabricate, the single web-girder should be used wherever possible.

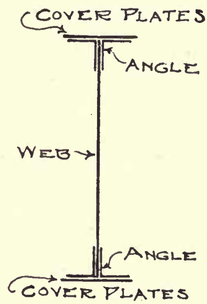


FIGURE 22

There are no two engineers who use exactly the same method in designing riveted girders, but the results given by the method employed in the following problems have always been satisfactory and can be used safely.

Given a girder with a clear span of 36 feet, a uniformly distributed load of 80 tons, and one concentrated load of 60 tons and another of 50 tons, respectively located 12 feet and 26 feet from the left support; the first step is the same as in the case of a simple beam, namely, to determine the reactions. The method is the same as used in Chapter III.

60 tons	× 12 feet	=	720 foot-tons.
80 "	× 18 "	=	1440 "
50 "	× 26 "	=	1300 "
190 tons			3460 foot tons.

$$3460 \text{ foot tons} \div 36 \text{ feet} = 96 \text{ tons} = R_2.$$

$$190 \text{ tons} - 96 \text{ tons} = 94 \text{ tons} = R_1.$$

The maximum shear is equal to the maximum reaction and is therefore 96 tons.

The depth of the girder is considered next. Usually the textbooks give the assumed depth of built-up girders as a certain ratio of the span. This ratio is usually assumed as one-ninth or one-tenth.

For a girder, 36 feet long, the depth would be about 3 feet 6 inches, or, 42 inches. This depth is the distance between the backs of the flange angles.

The difficulty arising from the use of the above ratio is that no account is taken of the loading on the girder. A formula, based on pure assumptions, but which gives satisfactory results, is often used. The formula is, $d = 2V/R$, in which V is the maximum shear and R is the shearing value of the rivets used in fabricating the girder. To use this formula it is necessary to assume the size of the rivets, but as a rule, three-quarter or seven-eighth inch rivets are used in all such work. Providing we assume the smaller diameter, we must find the value of three-quarter inch rivets in *double shear.*

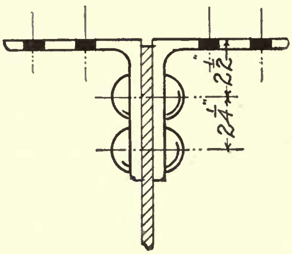


FIGURE 23

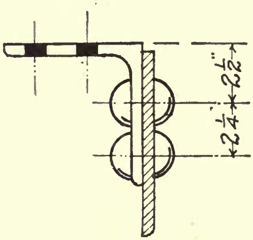


FIGURE 23a

When two angles are riveted to a plate, as shown in Fig. 23, then the rivets are said to be in double shear. When one angle is used (Fig. 23a), then the rivets are in single shear.

Under the heading of "Shearing Values for Rivets" in handbooks, these values are given for unit shearing stresses from 6,000 pounds per square inch to 10,000 pounds per square inch. In the 1909 edition of the Cambria Steel Company's book, page 316, the shearing value of a three-quarter inch rivet, having a unit shearing strength of 10,000 pounds per square inch, is given in double shear as 8,836 pounds, or, roughly, as 4.4 tons.

The vertical shear is 96 tons so, as $d = 2\ V/R$, $d = 2 \times 96/4.4 = 44$ inches.

This "d" is known as the *effective depth*, and is not the same as the one given by the use of the ratio. Fig. 24 shows a section of the girder, and between the rows of rivets in the flange angles, center lines are drawn. The distance between these center lines is "d."

To find the location of the center lines, it is necessary to find the distances from the back of each angle to the centers of the rivet holes. These distances are usually known as the "gauge." Fig. 23 gives the gauge for a 6-inch leg and the distance from the back of the angle to the center of the rows of rivets is $2\frac{1}{2}'' + 1\frac{1}{8}'' = 3\frac{5}{8}''$ as shown in Fig. 24. $3\frac{5}{8}'' + 44'' + 3\frac{5}{8}'' = 51\frac{1}{4}''$ or roughly 52 inches which equals the distance from the back to the back of the flange angles. This

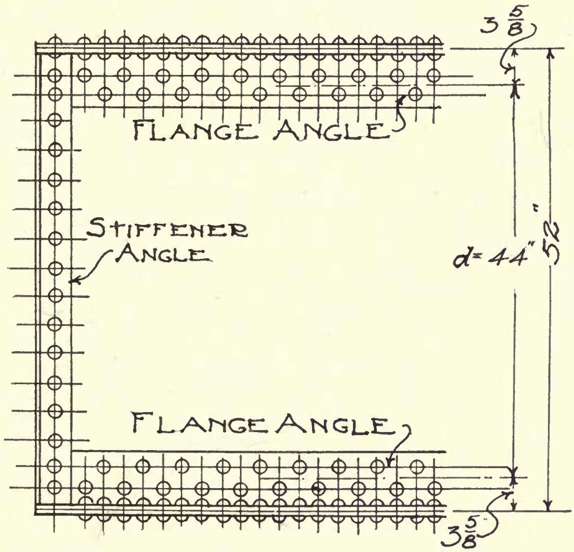


FIGURE 24

is taken as the depth of the girder, and the use of the formula always gives a greater depth than that obtained by the ratio.

In construction, the depth is often limited by conditions such as the thickness of floors or the available head room, but where it is possible to use a deep girder, the depth obtained by the above formula gives good results, as the deeper the girder the smaller the flange area. In this case a depth of 4 feet, or 48 inches, will be used.

The next step is the determination of the thickness of the web. This can be found directly from the table giving the shearing value of rivets, as this table also gives the bearing values of riveted plates. The shearing value of a three-quarter inch rivet, in double shear is 8,836 pounds. Following to the right, it is found that a $\frac{1}{4}$-inch plate has a bearing value of 3,750 pounds, a 9/16-inch plate has a bearing value of 8,438 pounds, and a $\frac{5}{8}$-inch plate has a value of 9,375 pounds. Under this last value a black line is drawn, showing that 9,375 pounds

is about the same as 8,836 pounds, the shearing value of the rivet. A 9/16-inch plate would give a value a little less than that of the rivet. If the web plate should be $\frac{5}{8}$-inch thick, there would be no more tendency for the plate to fail by bearing than for the rivet to fail by shearing.

The above process gives the thickness of the web plate, but this must be checked to determine whether the web will be strong enough to resist the shear. Flanges resist bending and webs resist shearing. The maximum shear is 96 tons. The area of the plate is $48'' \times \frac{5}{8}'' = 30$ square inches. Assuming a shearing value of 4.5 tons per square inch the value of the plate is given by $30 \times 4.5 = 135$ tons, which is considerably greater than necessary to withstand the shear.

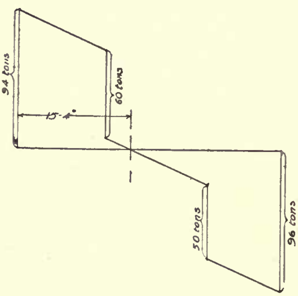


FIGURE 25

So far we have determined the depth of the girder and the thickness of the web plate. The next step is the determination of the flange members. The formula used in this case is $M = SAD$, a fairly easy one to remember. M equals the maximum bending moment, S equals the safe tensil or compressive stress of steel, A equals the area of the flange, and D equals the depth. To find A we must first find M. The shear diagram, shown in Fig. 25, gives the point of maximum bending moment as 15.3 feet from R_1. The method employed in drawing this shear diagram was explained in Chapter III.

To find the maximum bending moment at this point first find the upward moment caused by the reaction. 94 tons $\times$ 15.3 feet = 1438.2 foot-tons. The downward moments caused by the loading are 2.22 $\times\ 15.3\ \frac{15.3}{2} \times = 259.84$ foot-tons, and $60 \times 3.3 = 198$ foot-tons, or a total downward moment of 457.84 foot-tons. The total maximum bending moment at this point then is $1438.2 - 457.8 = 980$ foot-tons approximately.

Now, as $M = SAD$ and M equals 980 foot-tons, S in the case of riveted steel equals 7.0 tons,[1] and D equals 48 inches, A can be found. $A = \frac{M}{SD} = \frac{980}{4 \times 7.0} = 35$ square inches.

[1] The New York Building Department now will allow a safe working stress of 8 tons per square inch on net area.

This area is made up of two angles and cover plates, and to determine the size of these members some experimenting is required. Good practice requires that all members should have about the same thickness. It would be bad design to have an angle, 1 inch thick, riveted to a plate $\frac{3}{8}$ inch thick. As we have already assumed that one leg of each flange angle is 6 inches long we can assume that the length of the other leg is also 6 inches, and the thickness is the same as that of the web plate or $\frac{5}{8}$ inch. So the flange angles will be $6 \times 6 \times \frac{5}{8}$ inch.

Under the heading of "Properties of Standard Angles-Equal Legs" the area of a $6 \times 6 \times \frac{5}{8}$ inch angle is given as 7.11 square inches. Two angles would have an area of 14.22 square inches. The total area of the flange being 35 square inches, the area left to be made up by the cover plates will be $35 - 14.22 = 20.78$ square inches. The width of the cover plates can be taken as 14 inches, and 20.78 square inches divided by 14 inches will give the total thickness of cover plates as $1\frac{1}{2}$ inches. The thickness can be made up of three $\frac{1}{2}$-inch plates.

The bottom flange is in tension and a portion of the material is lost because of the rivet holes. Fig. 26 shows a portion of the flange and if the plates should fail by tension — pull a part — the failure would occur on either line *BB* or *AA*. It is practically certain that failure could not occur on line *CC*. There would be two rivet holes in the fractured section. For $\frac{3}{4}$-inch rivets, holes are punched $\frac{7}{8}$ inch in diameter. Each hole pierces three $\frac{1}{2}$-inch plates as well as an angle $\frac{5}{8}$ inch thick. So the area lost by each rivet hole will be ($1\frac{1}{2}$ inches + $\frac{5}{8}$ inch) $\times \frac{7}{8}$ inch = 1.86 square inches and two holes will cause a loss of 3.72 square inches.

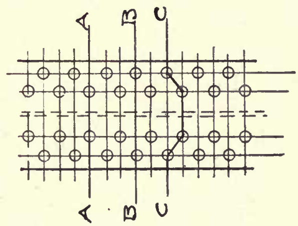


FIGURE 26

In the case of the upper flange 20.78 square inches had to be made up by plates, and as 3.72 square inches are lost by rivet holes in the bottom flange, $20.78 + 3.72 = 24.50$ square inches must be made up. The total thickness is given by $24.50 \div 14 = 1\frac{3}{4}$ inch, and the flange will have one cover plate $\frac{1}{2}$ inch thick and two $\frac{5}{8}$ inch thick. To facilitate the fabrication of the girder both flanges are made alike and the plates are cut the same length.

To determine the length of the cover plates the bending moment

diagram is made use of. Nothing has been said about the method of drawing bending moment diagrams, as they are seldom used except in such problems as this. Under the heading of "Bending Moments and Deflections of Beams of Uniform Sections" several types of diagrams are shown in the handbooks. When a beam is uniformly loaded the bending moment diagram takes the form of a parabola.

The method of drawing a parabola is simple, and as it is necessary to know this in order to draw the final diagram for the girder the process is worth mastering. The maximum bending moment for a uniform load is given by the formula $M = \frac{1}{8}\,Wl$. In case there is a uniform load of 80 tons, on a girder 36 feet long, the maximum bending moment is $\frac{1}{8} \times 80 \times 36 = 360 =$ foot-tons.

In Fig. 27 the line *ab* is laid off to represent 36 feet, for the purposes of demonstration, say at a scale of $\frac{1}{8}$-inch equals 1 foot. This

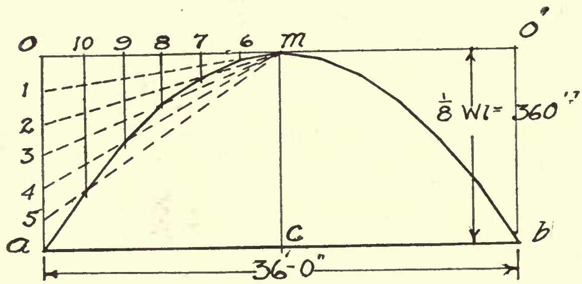


FIGURE 27

is the base of the parabola and is equal to the length of the girder. The line *cm* is laid off to represent the maximum bending moment. If an inch is considered as representing 200 foot-tons, 1.8 inches will represent 360 foot-tons. Draw *ao*, *oo'*, and *o'b* and divide *oa* into six parts. Also divide *om* into an equal *number* of parts, in this case six. Starting from *o* on *oa* mark the points 1, 2, 3, 4 and 5 and from *m* on *mo* mark the points 6, 7, 8, 9 and 10. From *m* draw radiating lines to 1, 2, 3, 4 and 5, and from 6 draw a vertical line to *m*1, from 7 to *m*2, and so until from 10 a vertical line intersects *m*5. These points of intersection are points on the parabola and by joining them the bending moment diagram for a uniform load of 80 tons on the girder is completed.

Again referring to the handbook, the bending moment diagram for a single concentrated load is seen to be a triangle, the base being

equal to the span and the altitude equal to the maximum bending moment. To determine the value of this maximum bending moment, due to the single concentrated load, the following method is employed.

In Fig. 28, a girder, with a span denoted by "l," is loaded with a single load "W" at a distance "a" from R_1 and "b" from R_2.

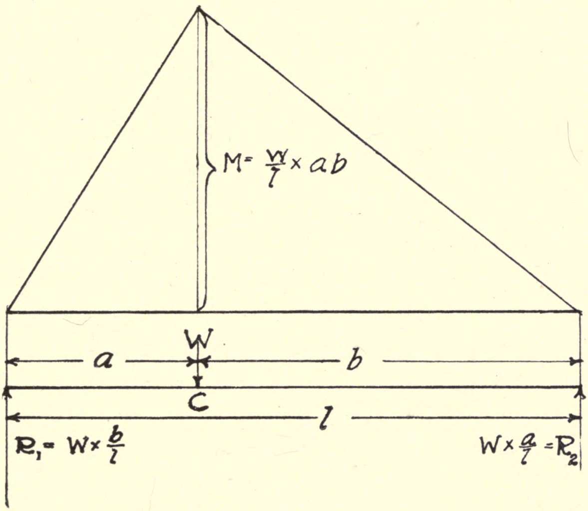


FIGURE 28

Then the downward moment around R_1 equals $W \times a$ and this equals the upward moment of $R_2 \times l$, or, $Wa = R_2 l$, R_2 then must equal $\frac{Wa}{l}$

In the same manner $R_1 = \frac{Wb}{l}$. The bending moment at c equals $R_1 \times a$ or $\frac{Wb}{l} \times a = \frac{W}{l} \times ab$. This checks with the moment of R_2 around c, or, $\frac{Wa}{l} \times b = \frac{W}{l} \times ab$.

In the case of a girder having a span of 36 feet and a load of 60 tons located 12 feet from R_1, the maximum moment caused by the load will be $\frac{60}{36} \times 12 \times 24 = 480$ foot-tons, and for a load of 50 tons, 10 feet from R_2 the maximum moment will be $\frac{50}{36} \times 10 \times 26 = 361.1$ foot-tons.

In Fig. 29 all the diagrams described above are shown. The two triangles represent the bending moments caused by each concentrated load. The parabola represents the bending due to the uniformly distributed load. The line *ABCDE* represents the *sum* of all the moments due to *all* the loads and is therefore the actual bending moment diagram. The point *B* is found by the addition of *OX*, *OY*, and *OZ*, or, in other words, $OB = OX + OY + OZ$. In the same manner all other points on the diagram are found. It will

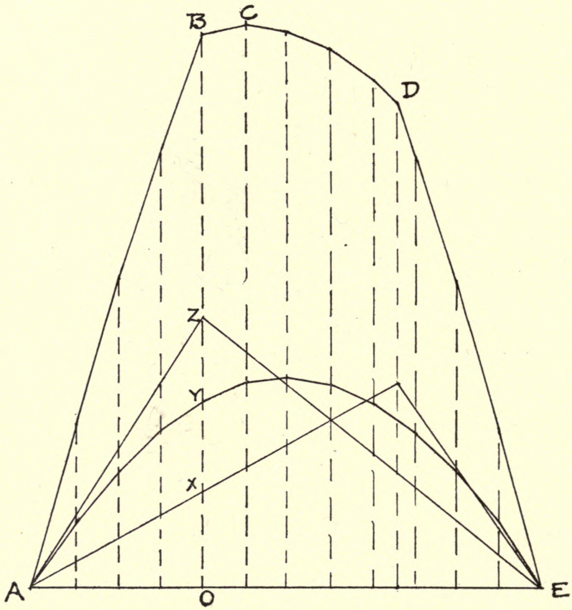


FIGURE 29

be found that the maximum bending occurs at *C* which is 15.3 feet from *A*.

Now the question naturally arises, what is the use of all this?

Disregarding all the more or less complicated formulas employed to prove that the following processes are correct the next step will be to find the length of the cover plates.

In Fig. 30, the bending moment diagram, already determined, is shown. The area of each flange is made of two angles — 6 inch $\times$ 6 inch $\times$ $\frac{5}{8}$ inch — having an area of 14.22 square inches, one plate — 14 inch $\times$ $\frac{1}{2}$ inch — having an area of 7.0 square inches, and two

plates — each 14 inches × $\frac{5}{8}$ inch,—having together an area of 17.5 square inches, making a total flange area of 38.72 square inches.

Draw through the point *e*, which is located at the point of maximum bending moment, a horizontal line *dd*. Between *RR* and *dd* lay off a line *AB*, at some convenient scale, having a length of 38.72 units. On this line lay off *AX*, having a length of 14.22 — the area

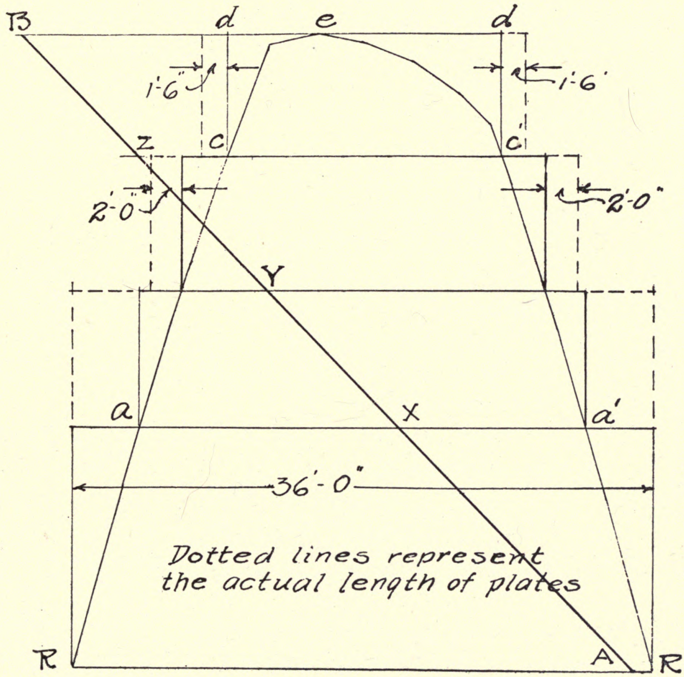


FIGURE 30

of the angles,— *XY* having a length of 8. 75— the area of one $\frac{5}{8}$-inch plate, *YZ*, the same length, and *ZB* equal to 7.0 units — the area of the $\frac{1}{2}$-inch plate. Through *X*, *Y* and *Z* draw horizontal lines. The line through *X* pierces the diagram at *a* and *a'*, and the length *aa'*, is the theoretical length of the $\frac{5}{8}$-inch plate. The length *bb'* is the theoretical length of the next $\frac{5}{8}$-inch plate, and *cc'* is the length of the $\frac{1}{2}$-inch plate. The angles of course run the full length of the girder. In actual fabrication the first $\frac{5}{8}$-inch plate is made to cover the total length of the girder. The bottom plate

is made 1 foot 6 inches longer on each end than is theoretically required to allow for riveting and the $\frac{5}{8}$-inch plate is made at least 2 feet longer on each end.

The reason for giving less projection for the bottom plate beyond that theoretically required, is because at one point only is this plate stressed to its limit, this point being where the shear is zero. At all other points in the plate there is a little more material than necessary.

FIGURE 31

The only other step to be taken in the design of a riveted girder, that an architect needs to know about, is the design of the stiffeners. At the points of support, and usually under each concentrated load, stiffeners are used. Where a heavy uniform load is carried by a girder stiffeners are riveted in such positions that the distance between them is no greater than the depth of the girder.

In Fig. 31 the diagram shows how buckling may occur in the web of the girder. The web plate being only $\frac{5}{8}$ inch thick might easily bend as shown, provided it is not braced. For the purposes of bracing angles are employed.

There are many formulas now being used to determine the size of these angles but the simple formula $P = SA$ is, in the opinion of the author, safe enough. P equals the load, S equals the safe compressive stress of riveted steel, and A equals the total area of the angles.

In the case of the girder P is the maximum shear — 96 tons — and S is 7.5 tons. A has to be determined. $96 = 7.5 \times A$, or $A = \frac{96}{7.5} = 12.6$ square inches. The outstanding leg of the angle can be no more than 5 inches or it would overlap the flange angle. A single 5 inch $\times$ $3\frac{1}{2}$ inch $\times$ $\frac{7}{8}$ inch angle will have an area of 6.68 square inches. Two angles will have about the area required.

Another method of determining the stiffeners is to consider the area of the outstanding leg of the stiffener angle only. The leg will be 5 inches long and S in this case is 20,000 pounds per square inch, or 10 tons. The thickness of the angle is found as follows. $96 = 2 \times 5 \times 10 \times t$ in which $t =$ the thickness.

$$t = \frac{96}{100} = \text{approximately } \frac{7}{8}''.$$

These stiffeners are riveted over the flange angles, and filler plates $\frac{5}{8}$ inch thick are riveted between the web plate and the stiffeners.

The spacing of rivets is usually determined by the conditions of fabrication. Any good firm of steel contractors will supply more than the necessary number of rivets and the architect will have no reason to bother about this consideration.

CHAPTER V

Columns. Formulas. Assumed section and safe stress per square inch. Dimensions. Moments of inertia. Radii of gyration. Actual safe working stress. Supporting value of section. Checks.

THE design of columns involves some new considerations, — such as the determination of the radius of gyration —which seem complicated and confusing at first glance, but which really are simple if attacked intelligently. In the first article the value of S — the safe working stress for steel — was given as sixteen thousand pounds per square inch. This value was obtained by crushing cylinders of steel in testing machines, and finding the force necessary to fracture the metal. The average crushing value of steel has been found to be 64,000 pounds per square inch, and, as a factor of safety of four is always used with this metal, the safe stress becomes $64,000 \div 4 = 16,000$ pounds per square inch.

If columns of steel were short in proportion to their sectional area they would act in the same manner as the blocks of steel crushed in the testing machines, but this is never the case. Not only does a column tend to fail by crushing, but also by bending. No one really knows just how the combination of crushing and bending stresses affects the material in a column. Formulas have been derived in which every theoretical consideration seems to have been accounted for, but these formulas have all had to be modified to agree with actual results obtained from experiments in which columns were subjected to crushing forces.

The first formula, of any kind, given in nearly every textbook on engineering is $P = SA$ in which P equals the compressive load, S equals the safe unit stress and A equals the area. If P were to be taken as 300 tons, and S as 8 tons per square inch, then A must equal $\frac{300}{8} = 37.5$ square inches as the area of a short block of steel supporting a load of 300 tons. If the block of steel were not short, but had a height comparatively great in relation to its sectional area, then it is obvious that S would become less, and in order to support the 300 tons, the area would have to be increased.

The general method of attacking a problem pertaining to the design of columns is to assume that a smaller value of S must be

taken. Once S is established the load is divided by it, and the cross-section area of the column is obtained. The real problem involved in column design is the determination of S.

The handbook published by the Cambria Steel Company gives a formula for S derived by Gordon. This formula gives the ultimate breaking strength per square inch of a column section as

$$S = \frac{50{,}000}{1 + \frac{(12L)^2}{36{,}000r^2}}$$

and if a factor of safety of 4 is used S becomes

$$S = \frac{12{,}500}{1 + \frac{(12L)^2}{36{,}000r^2}}$$

in which L is the length in feet and r is the "radius of gyration" in inches. Gordon's formula is the oldest and best known of all those used in modern practice and is found in the handbooks under the heading "Safe loads . . . for . . . Columns."

The formulas, giving S as required by the building laws of different cities, vary from Gordon's formula. The table of "Allowable Unit Stress and Loads" is not indexed but can be found in the 1909 edition of Cambria — on page 310. It gives S as required by the New York Building Code as $15{,}200 - 58\, l/r$, both l and r being measured in inches. This formula is much more simple than Gordon's.

To design a column to conform to the requirements of the New York Building Law, it is necessary to understand what l and r stand for. l is the length in inches of the unsupported length of the column, this distance usually being taken as the height from the top of the floor beam to the bottom of the floor beam directly above it. r is the *least radius of gyration* and this term needs some explaining.

As, in the second article, there was no definition of the moment of inertia given, the architect need not bother to find a definition of the least radius of gyration. The only important thing to remember is the formula $r^2 = I/a$ in which r is the radius of gyration, I is the moment of inertia, and a is the area of the cross section. To find r it is necessary to first find I.

Take a problem in which it is necessary to design a plate and

channel column, having a section strong enough to withstand a load of 300 tons, and having an unsupported length of fourteen feet. The first step is to assume a trial section.

It has been found that for heights from twelve to sixteen feet and average column sections, S is approximately 12,000 pounds or six tons per square inch. As the load (P) is 300 tons and S is 6 tons per square inch A must be $300 \div 6 = 50$ square inches.

Assuming that the architect desires that the column shall not be more than fourteen inches in any direction, this limits the designer to the use of twelve-inch channels and fourteen-inch plates. Good design usually results from having as nearly equal areas in the plates and channels as possible. To get an approximate area for each channel, divide 50 square inches by four and 12.5 square inches is the required area. A twelve-inch channel, weighing forty pounds per foot, will have an area of 11.76 square inches. This is the largest twelve-inch channel there is, and will have to be used in the design of the columns. Two channels will have an area of 23.52 square inches. $50.00 - 23.52 = 26.48$ square inches to be made up of plates. There will be two sets of plates, each having an area of $26.48 \div 2 = 13.24$ square inches. As the width of the plates is fourteen inches, the thickness must be $13.24 \div 14$, which equals roughly one inch. The *trial section* then will be composed of two twelve-inch, forty-pound channels and two fourteen by one-inch plates.

In order to find the moment of interia of this section, the location of the channels and plates with relation to the center lines must be established. Under the heading "Standard Spacing of Rivet and Bolt Holes" in the handbooks, the distance "*m*" from the back of the channel to the center line of the rivet hole in the flange is given for nearly all sizes of channels. The distance for a twelve-inch forty-pound channel in $2\frac{1}{4}''$. In the fabrication of a column the distance from the edge of the plate to the center of the rivet hole is usually one inch and a half. This makes the distance from the edge of the plate to the back of the channel $2\frac{1}{4}'' + 1\frac{1}{2}'' = 3\frac{3}{4}''$ (Fig. 32). The plate, being fourteen inches wide, the distance from the edge to the center is seven inches and the distance from the back of the channel to the center will be $7'' - 3\frac{3}{4}'' = 3\frac{1}{4}''$.

The moment of inertia shown in Fig. 32 around axis XX will be different from that around YY, and it will be necessary to find both.

In the handbooks, under the heading of "Properties of Standard Channels" the channels are shown to have one axis marked 1-1 and another marked 2-2. *I*, for axis 1-1 of a twelve-inch, forty-pound channel, is found to be 196.9. For two channels this moment of inertia is 393.8. To find the moment of inertia of the column section, the *I* for the plates around *XX* must be found and added

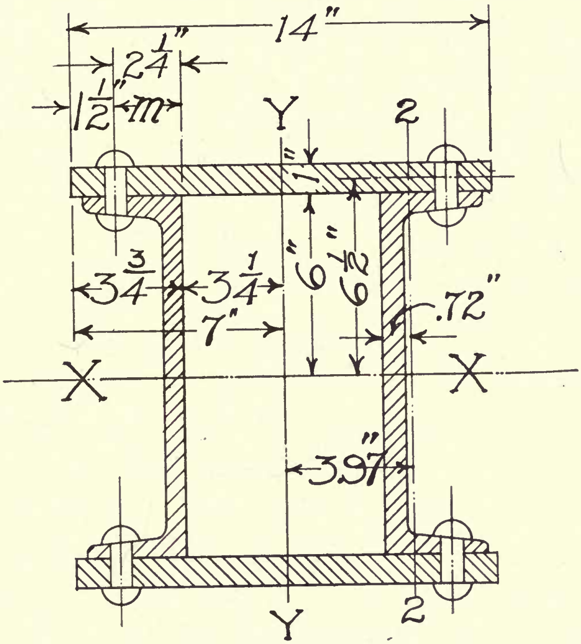


FIGURE 32

to that of the channels to give the *total* moment of inertia. In order to find this, the formula $I = I' + ah^2$ must be used. I' equals the moment of inertia of the plates, a equals their area, and h equals the distance from the axis XX to their center of gravity. In Fig. 32 h is found to be $6'' + \frac{1}{2}'' = 6\frac{1}{2}''$. a equals $14'' \times 1'' = 14$ square inches. The only other factor to be found is I' and this is determined by the formula $I = \frac{1}{12} \times bd^3$. $b = 14''$, $d = 1''$, so $I \times \frac{1}{12} \times 14 \times 1 \times 1 \times 1 = 1.16$. The moment of inertia of the plates around XX is given as $I = 1.16 + (14 \times 6\frac{1}{2} \times 6\frac{1}{2}) = 1.16 + 591.5 = 592.6$. As 1.16, or I', is such a small quantity in relation to 591.5, it is often disregarded by engineers. There are two sets of plates, so the mo-

ment of inertia of the two sets will be $592.6 \times 2 = 1185.2$. Add this to the I of the two channels and the total I for the section will be $I = 393 + 1152 = 1545$.

To find the moment of inertia around YY, the same method is employed. The I of one set of plates is given, as before, by the formula $I = \frac{1}{12} \times bd^3$, only, in this case, $b = 1''$ and $d = 14''$. $I = \frac{1}{12} \times 1 \times 14 \times 14 \times 14 = 229$. Two sets of plates will have a moment of inertia of 458. The I for the channels will be determined by the formula $I = I' + ah^2$. In the tables I' for a twelve-inch, forty-pound channel, around axis 2-2 is 6.63. The area of each channel is 11.76. h is the distance from axis 2-2 to the center line YY. The distance from 2-2 to the back of the channel is given in the handbook is .72″. We have found that the distance from the back of the channel to the center is 3.25 inches, so the distance from YY to 2-2 is $3.25'' + .72 = 3.97$ inches. To find the I of each channel around YY, simply substitute in the formula $I = 6.63 + 11.76 \times 3.97 \times 3.97 = 192$. Two channels will have a moment of inertia of 384 and this, added to the I of the plates, will give $384 + 458 = 842$.

The two moments of inertia are 1545 and 842. If one had equaled the other, the design would have been better, but the architectural requirements make this impossible and the column will have a greater tendency to fail around axis YY than around XX.

The only reason for finding I for the section is for the purpose of determining the least radius of gyration. As the area of the section also enters into the calculation this must be determined. The channels have together an area of 23.52 square inches, and the two sets of plates will have an area of 28 square inches, making a total area of 51.52 square inches. As $r^2 = \frac{I}{a}$, $r^2 = 16.4$, and r equals the square root of this or 4.05.

This value, 4.05, is the least radius of gyration and can be used to find the value of S in connection with the above problem. $S = 15{,}200 - 58\frac{l}{r}$ or $15{,}200 - 58 \times 14 \times 12 \div 4.05 = 12{,}800$ pounds per square inch approximately.

As the area of the section is 51.5 square inches, and the allowable bearing value for the section is 12,800 pounds per square inch, this gives the total strength of the columns as $12{,}800 \times 51.5 = 659{,}200$ pounds or about 330 tons. This is too large, and a reduction in the area of the column section must be made. Instead of using one-inch plates, seven-eighth-inch plates can be made use of.

As a check for this again find the moment of inertia around *YY*. The channels will give the same I as before — 384 — and, looking in the handbooks under the heading of "Moments of Inertia — Tables" the I for a plate $14'' \times \frac{7}{8}''$ is found to be 200.08. Two plates will have an I of 400.16. The total I for the section is $384 + 400 = 784$. The area is $23.52 + 24.50 = 48$ square inches — roughly — and $r^2 = \frac{784}{48} = 16.3$. $r = 4.03$. $S = 12{,}776$ and the strength of the section is $12{,}776 \times 48 = 613{,}248$ pounds or 307 tons which is safe.

If it is desired to use Gordon's formula, as given in the handbooks, it will be found that the heavier section will be required.

$$S = \frac{12{,}500}{1 + \frac{(12L)^2}{36{,}000r^2}}.$$

$$L = 14 \text{ and } r^2 = 16.4 \text{ so}$$

$$S = \frac{12{,}500}{1 + \frac{(168)^2}{36{,}000 \times 16.4}} = \frac{12{,}500}{1.048} = 11{,}920.$$

The area being 51.5 square inches, the strength of the section is $11{,}920 \times 51.5 = 615{,}000$ pounds or 307 tons.

In the Cambria handbook safe loads are given in thousands of pounds for various column sections. The plate and channels columns found in the tables have plates varying in thickness from $\frac{1}{4}''$ to $\frac{5}{8}''$ or from $\frac{3}{8}''$ to $\frac{3}{4}''$. In no case is there a heavy enough section having twelve-inch, forty-pound channels and fourteen-inch plates to carry the 300-ton load. By proportion, however, it is possible to check the result given above.

From the tables — page 270 in the 1909 edition of Cambria — it can be found that a column having a section made of two twelve-inch, forty-pound channels, two $\frac{1}{4}'' \times 14''$ plates, and having a clear height of fourteen feet, will support a load of 365,000 pounds. The same section having $\frac{1}{2}'' \times 14''$ plates will support a load of 448,000 pounds. The addition of $\frac{1}{4}''$ to the plates will give an increase of supporting power of 83,000 pounds. If this is true, an increase of $\frac{1}{2}''$ would give an increased supporting value of 166,000 pounds. If the section having $\frac{1}{2}''$ plates will support 448,000 pounds a section having $1''$ plates will support $448{,}000 + 166{,}000 = 614{,}000$ pounds or 307 tons. This checks with the results given above, but as this method is extremely rough, it should only be used in checking.

The values given in the handbooks for safe loads on columns do very well for light loads, but when heavy loads are encountered and the area of the column is limited, it is necessary to figure the sizes of the members in the cross section. However, if there were no limit, the sizes can be taken directly from the handbooks. It will be found that a column composed of two 15″ channels, weighing 45 pounds per foot, and two $17'' \times \frac{11}{16}''$ plates will support a load of 606,000 pounds for a clear height of 14 feet. Gordon's formula, as used in all these cases, gives a heavier section than is absolutely necessary and the New York Building Code formula gives more economical results.

In the cases mentioned in this article, plate and channel columns alone have been used. This is done because for a beginner the section given by the use of plates and channels has several advantages. It is almost a square section and therefore can be turned in such a direction as will suit the architectural requirements, without materially affecting the design. The riveting of plate and channel columns is very simple but connections are not made easily.

Columns are fabricated in two story lengths. The advantages of this practice are obvious — the frame of the building is made stiffer and the fabrication and erection is simplified. The loading on the column increases at each floor, and engineers usually figure the size of each two-story length so that it will withstand the heaviest load coming upon it. If the architect has a column schedule in his office, it would be well for him to consult this, noticing the loads brought to each column, the height of each length, and the areas of the sections.

In skeleton steel construction, the columns bring the loads of the building down to the foundations. Usually these loads are distributed over the foundations by means of grillage beams and the determination of the sizes of grillage beams will be taken up in the next chapter.

Further consideration of columns will be found in Chapter IX.

CHAPTER VI

Grillage beams. Failure by crushing. Assumed number of beams in lower layer. Calculated lengths. Size of concrete footing. Checks. Grillage for outside columns.

THE subject of grillage foundations embraces theoretical and practical considerations too numerous to be discussed within the limits of a single chapter. The best that can be done is to give a general description of this type of foundation and to show how engineers dispose of some of the conditions involved in its design.

The Gothic builders first worked on the principle of concentrating the weight of their edifices upon isolated supports. By means of balancing thrust with counter-thrust the weights of vaults, roofs, and walls were transferred to large piers, and the piers carried the loads to the masonry footings. In modern buildings steel columns serve the same purpose as the isolated piers, and the problem of foundation design arises in distributing the loads, brought to the footings, over an area large enough so that the weight per square foot will not exceed the bearing value of soil or concrete.

The average steel column is never much more than a foot and a half square, and carries from 300 to 500 tons. A square foot of reinforced concrete has a unit-bearing value of 35 tons, and soil can only support 4 tons per square foot. It can be seen from this that a load brought to the foundation by two and a quarter square feet of column, must be distributed over many more square feet of concrete or soil, or the footing will not bear up under the load. Grillage beams are used for this purpose of distribution.

Fig. 33 shows a perspective view of a grillage foundation. Under the column is a steel slab 17 inches wide, 3 inches thick and 1 foot 10 inches long. Under this are placed six 15-inch channels, arranged back to back in three pairs, and under the channels are placed seven I-beams which distribute the 500-ton load over the reinforced concrete.

In all previous articles, the only method used in determining the sizes of beams to withstand certain loads was to supply a beam strong enough to resist the tendency to fail by bending. Although it is always necessary to check the sizes of grillage beams, to deter-

mine their ability to resist bending, other considerations, such as crushing of the web, govern the design.

As in the case of column design, it is necessary to make trials in order to determine the actual sizes of the grillage beams, and the general dimensions of the footings. The dimensions of the slab are governed by those of the column. A column, made of 12-inch channels and 16-inch plates, with 4-inch by 6-inch clip angles, requires a

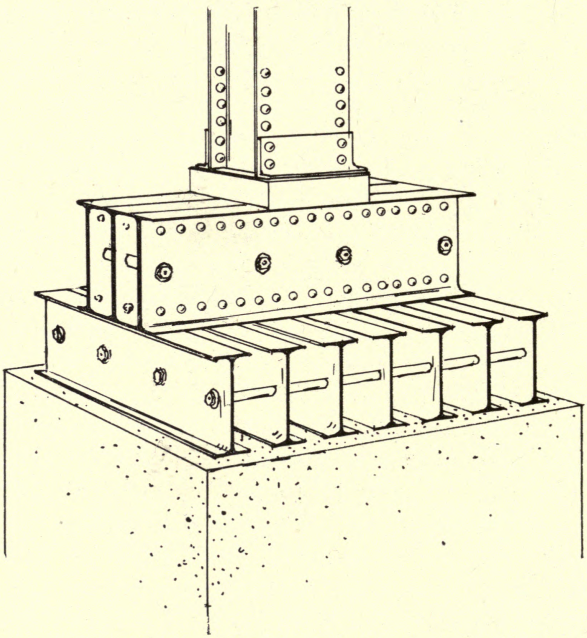


FIGURE 33

plate 17 inches wide by 1 foot 10 inches long. The method of determining the thickness of the plate will be taken up later.

When the foundations rest upon bed rock, the outside dimensions of the footing are governed by the bearing power of the reinforced concrete. The New York Building Department allows a pressure of one quarter of a ton (500 lbs.) per square inch on that reinforced concrete which is in *direct contact* with the steel. In other words, the portion of concrete included between the flanges of the lower layer of beams (*x* — Fig. 34) cannot be considered as supporting any portion of the 500 tons. For this reason it is necessary to use a

large number of light beams, having comparatively large flanges, instead of a few heavy beams with comparatively small flanges.

As a trial seven twelve-inch, thirty-five-pound, I-beams will be used in the lower layer. These beams have flanges 5.09 inches wide and a portion of one beam, one foot long, will have an area of $12'' \times 5.09'' = 61$ square inches. Seven beams will have $61 \times 7 = 427$ square inches of flange area bearing on the concrete per lineal foot of beam. As each square inch of concrete will sup-

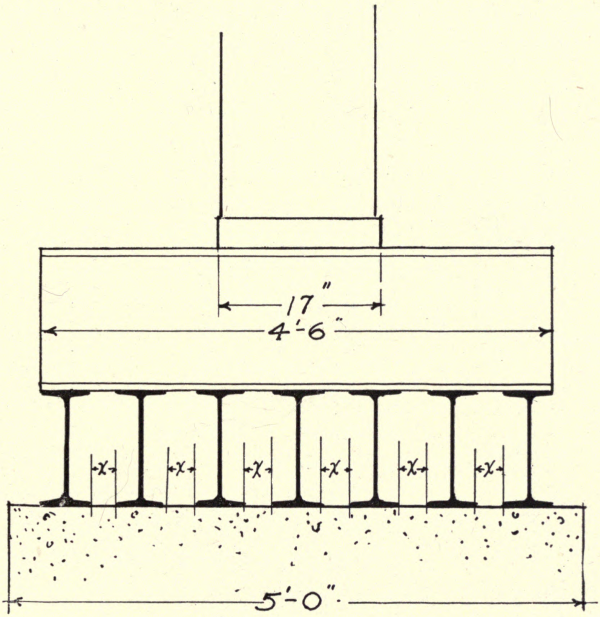


FIGURE 34

port one quarter of a ton, 427 square inches will support $427 \div 4 = 107$ tons. There are 500 tons to be carried so $500 \div 107 = 4.6$ feet of beams will be necessary. To give an even figure these beams will be made four feet six inches long.

It is usually considered good practice to have the footing square so the upper layer of beams will be made four feet six inches long also. The concrete should project at least three inches beyond the ends of the beams, so the outside dimensions of the footing will be five feet by five feet.

We now have assumed the area of the slab and the area of the concrete footing and have assumed the sizes of beams in the lower

layer. For the given conditions it is necessary to determine the sizes of beams in the upper layer. These beams will have to withstand a load of 500 tons coming down on 17 inches of their length. On account of the dimensions of the slab there can be only three beams, so each beam will have to support $500 \div 3 = 166.6$ tons. This load will tend to *crush the web* of the beam, and we must supply a beam strong enough to resist this crushing. The number of square inches of steel in the web of each beam under the slab will be given by $17'' \times t$, in which "t" is the thickness of the web. If each square inch will support 8 tons—the safe crushing value of steel—the total bearing value in tons of each web is $17 \times t \times 8$. This, of course, must equal 166.6, or, $17 \times t \times 8 = 166.6$. t must equal $166.6 \div 136 = 1.2$ inches. There is no I-beam having a web as thick as this, so in place of a single I-beam, two 15-inch, 45-pound channels riveted back to back are used, the two webs having a combined thickness of 1.24 inches. The upper tier of beams will then consist of six 15-inch, 45-pound channels, 4 feet 6 inches long. The only other dimension to be found in this trial footing is the thickness of the plate.

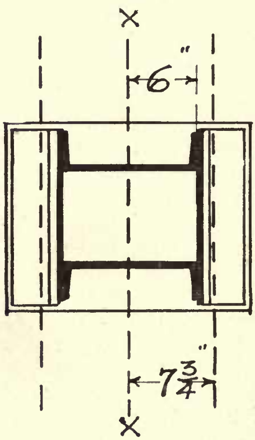


FIGURE 35

Fig. 35 shows a plan of the column and plate. The axis XX divides the column into two equal parts and a load of 250 tons is brought down on the right, and an equal one on the left, of this axis. The neutral axis of the right section of the column is assumed to be 6 inches from XX and the load of 250 tons will be concentrated along this line. The center lines of the beams under the slab are shown by dotted lines $7\frac{3}{4}$ inches apart. The upward load upon the slab exerted by each beam will be $500 \div 3 = 166.6$ tons. The upward moment will be $166.6 \times 7\frac{3}{4} = 1291$ inch-tons. The downward moment will be $250 \times 6 = 1{,}500$ inch-tons. The difference is $1500 - 1291 = 209$ inch-tons as the bending moment set up in the slab.

Referring to the first chapter we find that $M = S\, I/c$. $S = 8$ tons, and $I/c = 1/6\; bd^2$. In this case $b = 17$ inches, and the problem arises in determining d. $209 = 8 \times \frac{1}{6} \times 17 \times d^2$. $d^2 = 9$, so $d = 3$ inches. This gives us all the dimensions of the trial footing — the size of the plate, the sizes of the beams in the upper and lower layer,

and the dimensions of the concrete pier. It is necessary to check the beams, however, to determine their ability to resist bending and shearing stresses.

In cases where bending has been considered, the formula $M = \frac{1}{8}Wl$ has always been used. A modification of the formula is used in the case of grillage beams. The formula takes the form $M = \frac{1}{8}W\ (l - a)$ in which $(l - a)$ is the overhanging length of the beams. In Fig. 34, "a" is the length of the slab or 17 inches, and "l" is the total

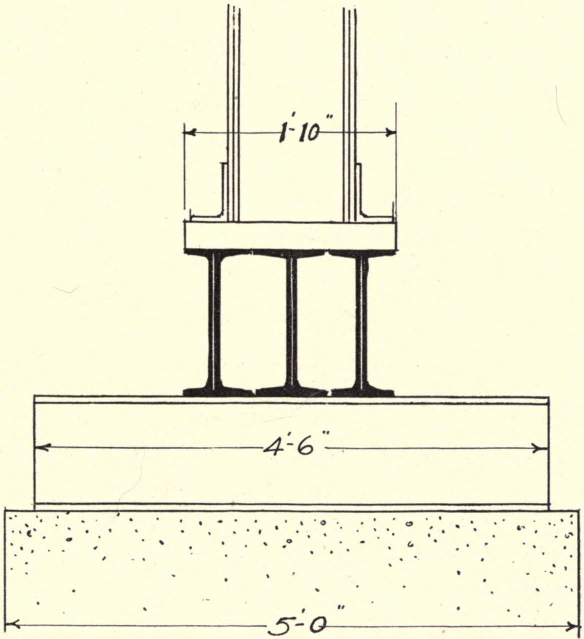


FIGURE 36

length of the channels or 4 feet 6 inches. $(l - a)$ equals 4 feet 6 inches minus 1 foot 5 inches or 3 feet and 1 inch.

If $M = \frac{1}{8}\ W\ (l - a)$ then in the case of the upper beams $M = \frac{1}{8} \times 500 \times 3.1 = 194$ foot-tons. The section modulus is found when foot-tons are used by multiplying the maximum bending moment by $1\frac{1}{2}$, as explained in chapter III. $194 \times \frac{3}{2} = 291 = I/c$. This is the combined I/c for the six channels and each channel will have to have a section modulus equal to or greater than $291 \div 6 = 49$. This is less than the I/c for a 15-inch, 40-pound channel, which is given as 50, so the channels are safe as far as bending is concerned.

The only other way in which failure can occur is by shearing the ends of the channels off at the edges of the plate. (Fig. 34.) The projection of the channels beyond each side of the plate is approximately 1′-6″. This is actually one third of the total length of the channels. If the channels must support 500 tons, each projecting length must support $500 \div 3 = 166.6$ tons. Each channel will then have a maximum shear of $166.6 \div 6 = 27.7$ tons. The area of the cross section of a 15-inch, 45-pound channel is 13.24 square inches, and the shear per square inch on the section is $27.77 \div 13.24 = 2$ tons, approximately. This is well within the safe shearing value of steel of 4.5 tons per square inch.

We now know that the channels in the upper layer are safe and we must check the sizes of the I-beams in the lower tier. Fig. 36 shows the location of the channels and the slab in relation to the lower beams. "a" in this case is 1 foot 10 inches, and $(l - a)$ must equal 4 feet 6 inches minus 1 foot 10 inches or 2 feet 8 inches. $I/c = \frac{3}{2} \times \frac{1}{8} \times 500 \times 2.66 = 250$ for seven beams. $250 \div 7 = 35$. The I/c for a twelve inch, thirty-five pound I-beam is 38 so these beams will not fail by bending.

These beams might fail by crushing and shearing, however, and it is necessary to determine whether this is the case or not. The projecting ends of the beams beyond the channels are 1 foot 4 inches long and we will be safe in assuming that the total shearing stress is 150 tons. As there are seven beams, the shear per beam will be $150 \div 7 = 21.4$ tons. The area of the cross section of twelve inch, thirty-five pound I-beam is 10.29 square inches. So the shear per square inch will be $21.4 \div 10.29 = 2$ tons approximately which is safe.

The section of the web of one beam directly under the slab has an area of 22 inches long by .44 inches thick or 9.8 square inches. Seven beams will have a total area of $9.8 \times 7 = 68.6$ square inches. As each square inch will withstand a pressure of 8 tons, 68.6 square inches will support $68.6 \times 8 = 548.8$ tons, so the beams will safely support the 500-ton load.

The trial footing, therefore, is safe in every particular.

The conditions accounted for so far, have been for footings where it was possible to have the concrete rest on bed rock, and where there was no limit to the dimensions of the footing in any direction. This is only one of many conditions that may arise. In case the column is an outside one and placed about two feet back of the building line it would be impossible to have a base five feet square. This condi-

tion is shown in Fig. 37, and the footing should be longer than it is broad.

In this case the column having a heavy wall load to support will have a load of 1,061 tons. The size of the footing will be considered first. As the column center line is two feet back of the building line the total width of the footing cannot be more than four feet or forty-eight inches. If five beams are used in the lower tier then there will be four spaces between them, and a slight over-hang on each end, so the beams will be 9″ on centers. The average width for heavy flanges is about seven inches so a foot of beam will have $12 \times 7 = 84$ square inches of flange bearing on the concrete, 5 beams will have

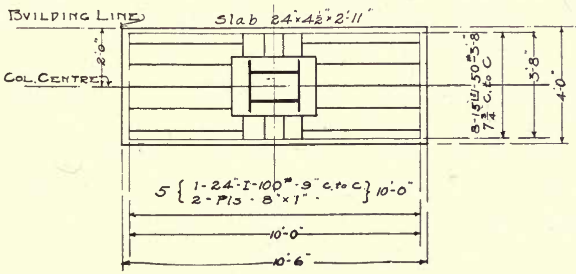


FIGURE 37

420 square inches. Each square inch of concrete will stand one-quarter of a ton so $420 \div 4 = 105$ tons per foot of beam. $1{,}061 \div 105 = 10$ feet. In the trial footing the beams in the upper tier will be taken as 4 feet long, and those in the lower tier will be 10 feet long. The conditions of fabrication will make the slab 24 inches wide by 2 feet 11 inches long.

The next step is to determine the sizes of the beams that make up the footing. The four beams in the upper layer are more apt to fail by crushing than by bending on account of their short length. The slab covers 2 feet of the beam so the thickness of the web must be found as follows: $1{,}061 = 24 \times t \times 4 \times 8$ or $t = 1061/768 = 1.4$ inches, so, for resistance to crushing, eight 15-inch channels weighing 50 pounds per foot must be used. The shear is one-quarter of the total load as the projecting length of the channels on either side of the slab is one-quarter of the total length. $1{,}061 \div 4 = 265$ tons. In this case, because some engineers figure that only the web should resist the shear, we will determine what the shear per square inch on the web of the channels will be. The total area of the web is

$8 \times 15 \times .72 = 86$ square inches $= 265 \div 86 = 3$ tons per square inch. The beams are safe from failure by shearing.

Failure by bending will be considered next. $I/c = 1\frac{1}{2}M = 1\frac{1}{2} \times \frac{1}{8} \times W(l - a) = 3/2 \times \frac{1}{8} \times 1{,}061 \times 2 = 397$. This is the combined section modulus for eight channels. I/c for one channel is $397 \div 8 = 50$. The section modulus of a 15-inch, 50-pound channel is 53.7 so the upper layer of beams is safe in every respect.

It would seem that the long beams in the lower tier would have a tendency to fail by bending. $(l - a)$ in this case equals 10 feet minus 3 feet, or 7 feet. The total I/c will equal $3/2 \times \frac{1}{8} \times 1{,}061 \times 7 = 1{,}393$. The section modulus for each beam is $1{,}393 \div 5 = 278$.

There is no standard beam that is heavy enough to withstand this bending, so a heavy beam reinforced with plates on the top and bottom flanges will be necessary. In case 24-inch, 100-pound I-beams are used, these plates will be one inch thick. The method of determining this thickness will be taken up in a later chapter. To resist crushing the web must have a thickness of $1{,}061 \div 36 \times 5 \times 8 = .73$ and the web of a 24-inch, 100-pound beam will be large enough. These beams should also be checked for shear but, as we have encountered no tendency toward failure in this direction, it can be assumed that there is none in this case.

When columns rest on soil foundations other problems arise, and, when the dimensions of the footing are limited in two directions, as in the case of a corner column, still further considerations have to be taken care of. These complications will be dealt with in the next chapter.

CHAPTER VII

Grillage foundations on soil. Size of concrete footing. Length of beams. Maximum bending moment and section modulus of upper beams. Tests for shearing and crushing. Design of lower layer of beams. Footings for corner columns. Cantilever beams.

WHEN column loads have to be distributed over soil the methods employed in figuring the footings are the same as in Chapter VI, but, as soil will only withstand a compressive force of four tons per square foot, the results differ from those of the preceding chapter. In the case where there was a bearing on rock, the failure of the grillage might have occurred in the concrete under the steel, the concrete being weaker than the rock. For this reason it was necessary to reinforce the concrete and to assume that only the concrete in direct contact with the steel had any bearing value. These precautions will not be necessary when the bearing is on soil as the earth will settle before the concrete will fail. In Chapter VI, the first consideration was the size of the concrete slab. In the present case it is the area of the soil over which the concrete slab is spread that must be determined first.

Given a column load of 400 tons, and the safe bearing value of soil as 4 tons per square foot, the area over which the load is to be distributed is $400 \div 4 = 100$ square feet. If the footing is made square the outside dimensions will be 10 feet by 10 feet. (Fig. 38.)

Allowing a projection of concrete of 4 inches on all sides the length of the beams in both layers will be 10 feet, minus 8 inches, or 9 feet 4 inches. (9.33 feet.)

The steel slab under the column will be made 3 feet square, and, using the formula for bending given in Chapter VI, the maximum bending moment for the beams in the upper layer will be

$$M = \tfrac{1}{8} \times 400 \times (l - a), \text{ or, } 50 \times 6.33 = 316 \text{ foot-tons.}$$

The section modulus is $316 \times 1\tfrac{1}{2} = 474$. It is possible to place three beams under the slab and the sections modulus for each beam will be $474 \div 3 = 158$. Three 24-inch, 80-pound I-beams will be sufficiently strong to withstand the tendency to fail by bending.

The beams project on each side beyond the slab approximately one third of their total length and the maximum shear will be

$$\tfrac{1}{3} \times 400 = 133 \text{ tons.}$$

The web of a 24-inch, 80-pound I-beam is $\frac{1}{2}$ inch thick, and the *area* of the web is $24 \times \frac{1}{2} = 12$ square inches. Three beams will have a total web area of $12 \times 3 = 36$ square inches. As the total shear is 133 tons, the shear per square inch of web will be $133 \div 36 = 3.66$ tons. The Building Department allows a shear of 4.5 tons on steel and the beams are safe as far as failure by shearing is concerned.

The only other method of failure to be considered, for the beams on the top tier, is the *crushing* of the *web*. The slab extends over a

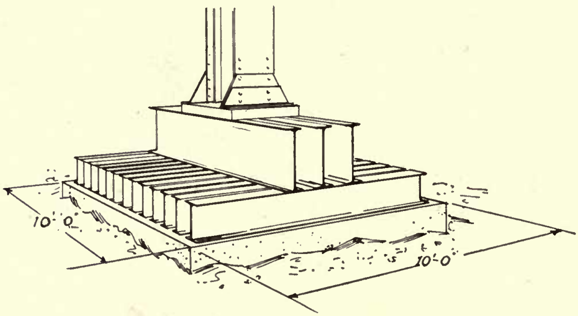


FIGURE 38

length of three feet and the webs are one-half an inch thick so the area of the web under the plate is $36 \times \frac{1}{2} \times 3 = 54$ square inches. The load is 400 tons, and the compression per square inch is $400 \div 54 = 7.4$ tons, which is safe. The upper layer of beams is strong enough in every particular.

In the lower layer the beams should be placed so that there will be comparatively little space between the adjacent flanges. If twelve beams are used, spaced $9\frac{5}{8}$ inches on centers, they will practically cover the concrete. The total section modulus for all of these beams will be the same as for those in the upper layer, as W and $(l - a)$ remains the same. This should be checked. In order to find I/c for a single beam, simply divide 474 by 12 and the result—39.5—will be the section modulus. The lower tier of beams will consist of 12 12-inch, 40-pound I-beams, 9 feet 4 inches long and spaced $9\frac{5}{8}$ inches on centers. By the methods already employed these

beams should be checked for shearing and crushing of the web. They will be found to be safe.

Fig. 39 shows a condition in which the corner column is so placed that a square footing cannot be used as two sides will be outside the building lines. As this is not allowed in most cities, a type of foundation must be employed somewhat different from any we have considered so far.

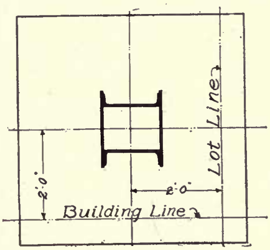


FIGURE 39

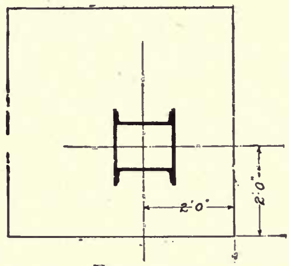


FIGURE 39a

If the footing were spread, as shown in Fig. 39a, so as to throw the center of the column off the center of the concrete base the tendency would be to make the footing settle unevenly. This tendency toward uneven settlement can be illustrated by considering a row boat in which the only occupant is located in the bow. The bow will sink slightly in the water and the stern will rise. If another person should be placed in the stern the two occupants would counter-balance each other and the boat would set evenly. In like manner, if the footing of the corner column were extended far enough to the left to allow the next outside column to rest upon it, the second column would act as a counter-balance for the corner one, and, provided the dimensions of the footing were properly proportioned, there would be no more tendency to settle in one direction than in another.

Fig. 40 shows a diagrammatic plan and elevation of this condition. Column No. 1 is the corner column, having a load of 650 tons, and which is located 2 feet back of the building lines. Column No. 2 carries a load of 600 tons and is in line with the corner column. The distance between No. 1 and No. 2 is 20 feet. Under the two columns two girders, *G*1 and *G*2, are placed and I-beams are distributed under the girders to carry the loads to the concrete footings. These footings rest upon rock.

If the architect understands clearly the method of solving the conditions referred to in Chapter VI, the only new problem to be solved is the position of the I-beams and concrete footings. First, it is necessary to assume the size of the I-beams and determine their length and numbers. As 12-inch, $31\frac{1}{2}$-pound I-beams have proved

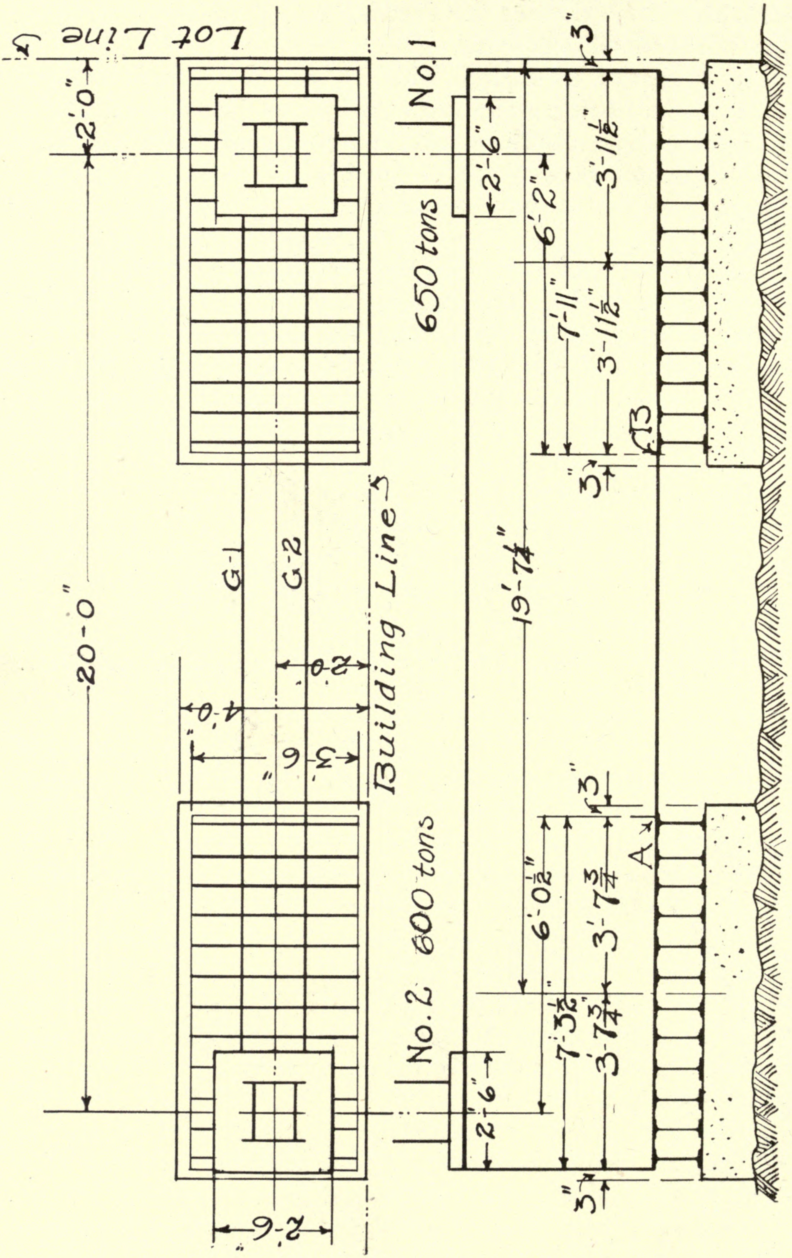

Lot Line
Building Line
No. 1
650 tons
No. 2
600 tons
G-1
G-2
20-0"
2'-0"
4'-0"
3'-6"
2'-0"
2'-6"
2'-6"
6'-2"
7'-11"
3'-11½"
3'-11½"
19'-7¼"
6'-0½"
7'-3½"
3'-7¾"
3'-7¾"
3"
A
B

FIGURE 40

satisfactory for most conditions imposed upon the lower layer of grillage beams, it will be *assumed* that they will be satisfactory in this case. The length is governed by the dimensions of the concrete. As the center of the footing is 2 feet back of the building line, the width of the footing can only be twice this or 4 feet. Allowing an overhang of 3 inches of concrete beyond the ends of the beams, the beams themselves can only be 3 feet 6 inches long. The flanges of 12 inch, $31\frac{1}{2}$-pound I-beams are 5 inches wide, and, as the beams are 42 inches long, there will be $42 \times 5 = 210$ square inches of flange bearing on the concrete for each beam. In Chapter VI the reinforced concrete in direct contact with the steel was considered as having a bearing power of one-quarter of a ton (500 pounds) per square inch. The concrete under each beam will support

$$210 \div 4 = 52.25 \text{ tons.}$$

Column No. 1 has a load of 650 tons. The number of beams under this column will be determined by dividing the load by 52.25. $650 \div 52.25 = 13$ approximately. There will be 13 beams under the corner column. In like manner there will be 12 beams under column No. 2. If the beams are placed $7\frac{1}{2}$ inches on centers the set under No. 1 will spread over an area of concrete of $(12 \times 7\frac{1}{2}) + 5 = 95$ inches. The 5 inches added to 90 is the width of a single flange. Ninety-five inches equal 7 feet 11 inches. (Fig. 41.) The I-beams under No. 2 will cover 7 feet $3\frac{1}{2}$ inches.

So far the considerations involved have been exactly the same as those mentioned in Chapter VI. The next step, however, is a new one and involves the consideration of moments. The architect should thoroughly understand what is meant by the term *moment* and in case there is any doubt on the subject he should look over Chapter I again. A center of moments can be taken at any point, and, for conditions of equilibrium, the sum of all moments around this center must equal zero. In other words, the upward moments must equal the downward moments around this center.

For the given conditions, the center of moments will be taken at the building line. Column No. 1 has a load of 650 tons, and the distance from its center to the building line is 2 feet, so the moment around the assumed center is $650 \times 2 = 1{,}300$ foot-tons. The second column has a moment of $600 \times 22 = 13{,}200$ foot-tons. The total *downward* moment is $1{,}300 + 13{,}200 = 14{,}500$ foot-tons. To counteract this there must be an *upward* moment equal to it and the two

sets of I-beams may be considered as exerting *upward* forces which will cause the upward moments. Each force will be equal to that brought down by the column under which each set of I-beams is placed, and the forces will be considered as acting at their respective centers of gravity.

The beams under column No. 1 will be placed as near the building line as possible, but, as there should be a projection of 3 inches of concrete beyond the steel, the center of the set will be (7′-11″ ÷ 2) + 3 = 4′-2½″ from the building line. The upward moment caused by the footing will be 650 × 4.20 = 2735 foot-tons. Subtracting this from 14,500 foot-tons leaves 11,764 foot-tons to be taken up by the second set of beams. As this moment must equal a force multiplied by a

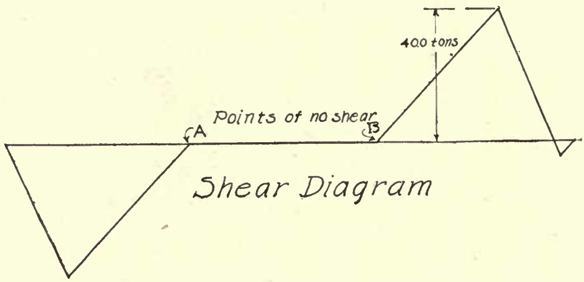


FIGURE 41

distance, and the force (600 tons) is known, the question is to determine the distance. Let X equal this distance. Then

$$600 \times X = 11{,}764, \text{ or } X = 11{,}764 \div 600 = 19.60.$$

Fig. 42 shows a diagram representing these conditions — the two columns acting downward and the two footings acting upward with their respective lever arms.

We now have the size, number and location of the I-beams in the trial footing. From knowledge which has already been acquired it is possible to lay out the beams as shown in Fig. 40.

The next step is the determination of the members in the girders. The two slabs, under the columns, are each 2 feet 6 inches long. The sets of I-beams will be assumed to act as uniform loads extending over 7 feet 11 inches and 7 feet 5 inches respectively. The shear diagram is shown in Fig. 41, and, as all the loads are uniformly distributed, the shear is represented as varying as a series of sloping lines. In Chapter III the statement was made that a beam will ail by bending at the point where the shear is zero. From the dia-

grams it can be seen that the shear is zero at any point between A and B. To find the bending moment at A we have two forces to account for — the 600-ton load of the column acting downward and the 600-ton load of the footing acting upward. Both loads act through their centers of gravity. The moment of the column load is 600 tons × 6.04 feet = 3624 foot-tons, and the moment of the footing is

$$600 \times 3.65 = 2{,}190 \text{ foot-tons.}$$

The difference is 1434 foot-tons. In like manner, the moment around B is $650 \times (6'\text{-}2'' - 3'\text{-}11\frac{1}{2}'') = 650 \times 2.208 = 1{,}435$ foot-tons. It will be noticed that both moments are approximately equal and

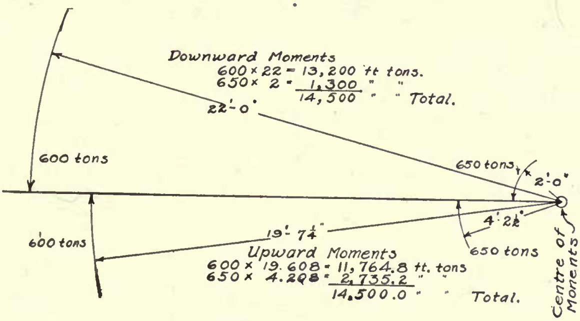


FIGURE 42

also that the same results may be obtained by multiplying the column load by the distance from the center of the footing; thus, 600 tons × 2.39 = 1434 foot-tons.

From the shear diagrams the maximum shear can be obtained. This is found at the point C — the edge of the slab under column No. 1 — and equals approximately 400 tons.

Two riveted girders, made of plates and angles, can be designed according to the principles mentioned in Chapter IV. The shear for two girders is 400 tons and for one is 200 tons. To use the formula for the depth of the girder given in the former article will give an excessive depth as the shear is large in proportion to the maximum bending moment. The most satisfactory method will be to assume the depth as some convenient figure — 4 feet — and design the girder for this depth.

Another method is to select the proper riveted girder from those given in the Carnegie "Pocket Companion." On pages from 235 to 250 are given lists of plate girders the sizes of which are determined

by the section modulus of each girder. In the present case the maximum bending moment for two girders is 1,435 foot-tons and the section modulus is $1{,}435 \times 1\frac{1}{2} = 2{,}152$. For one girder the section modulus is one-half this or 1,076. On page 249 of the Pocket Companion, a girder will be found having an I/c of 1,083 which is the nearest to the figure given above. These girders will be made of a 48-inch by $\frac{1}{2}$-inch web plate, four $6 \times 6 \times \frac{1}{2}$-inch flange angles, and two $14 \times \frac{5}{8}$-inch flange plates. In using such a girder it must be remembered that the safe working stress used as a basis for its design is taken as 8 tons. Some building departments require that for riveted steel this stress should be 7.0 tons. The girder should also be stiffened by stiffener angles directly under the columns and at

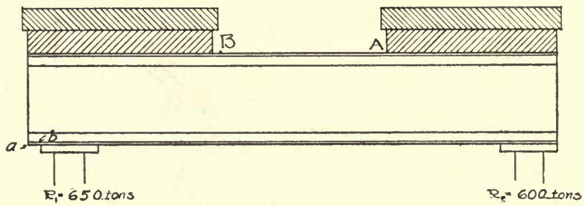


FIGURE 43

the points of maximum shear. The method of designing such stiffeners would be the same as used in Chapter IV.

So far the I-beams in the lower layer have not been checked for bending, but if $(l - a)$ is 1 foot and the load on twelve I-beams is 650 tons it will be found that they are safe. This is also true regarding failure by shearing and crushing.

Referring again to the Pocket Companion, on pages 224-25-26-27 and 28, it will be found that the problem of grillage beam design is dealt with. In these cases the footings are on soil and for a short and comprehensive treatment of the subject it is probably the best published.

For reasons, unknown to an engineer, architects often refer to such girders as designed in this article as cantilever girders. As a matter of fact only a small portion acts as a cantilever so the bending moments and shear are determined in exactly the same manner as if the girders were simple beams. Imagine Fig. 40 turned upside down as shown in Fig. 43. Now the column loads act upward and the uniform loads of the footings act downward. Only the portion *ab* overhangs the support and acts as a cantilever. Because *ab* is so small, the bending set up in the girders, due to this projection, is not

enough to influence the actual design of the girders as the tendency toward bending at the points A and B is so much greater.

As an actual example of a simple cantilever beam note the condition shown in Fig. 44. Here we have a load of 15 tons located 8 feet from R_2 and the portion of the beam projecting to the right of R_2 acts as a cantilever.

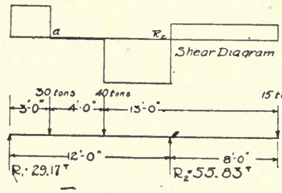


FIGURE 44

The design of such a beam is simple provided the architect does not set difficulties in his own path. To find the reactions the same method is employed as in the case of a simple beam. Taking R_1 as the center determine the downward moments around this center.

30 tons × 3 feet = 90 foot-tons.
40 " × 7 " = 280 "
15 " × 20 " = 300 "
Totals 85 tons 670 foot-tons.

670 foot-tons ÷ 12 feet = 55.83 tons = R_2.
85 tons − 55.83 tons = 29.17 tons = R_1.

The shear diagram is shown in Fig. 44 and from this it is apparent that there are *two points* of no shear. In other words there are two points at which the beam might fail by bending. It will be necessary to find the bending moment at each point to determine which is the greater. The one at a is 29.17 × 3 = 87.51 foot-tons, and the one at R_2 is 15 × 8 = 120 foot-tons. It is obvious that, if the beam should fail by bending, this failure would occur at R_2. As the bending moment at this point is 120 foot-tons the section modulus is found by multiplying 120 by $1\frac{1}{2}$, which gives an I/c of 180. A 24-inch, 85-pound, I-beam will be found to have this section modulus.

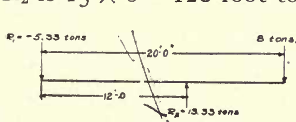


FIGURE 45

For the conditions shown in Fig. 45 a *downward* or *negative* reaction will occur at R_1. Taking R_1 as a center of moments and finding R_2; 8 tons × 20 feet = 160 foot-tons. One hundred and sixty foot-tons ÷ 12 feet = 13.33 tons = R_2. To find R_1 subtract R_2 from 8 tons, or, 8 tons − 13.33 tons = − 5.33 tons = R_1. The minus sign before 5.33 indicates that R_1 must act in the opposite direction from R_2.

CHAPTER VIII

Steel Framing Plan. Record of calculations. "Filling-in" beams. Girders. Trial calculations.

THE general principles outlined in the foregoing chapters can be applied to nearly all conditions encountered in designing the steel for a building. Just how to apply such knowledge, however, is sometimes a question. There are certain methods employed by an engineer which are extremely simple in themselves, but which are necessary if records are to be kept of the calculations, and which also facilitate the work. The architect who may know exactly how to determine the maximum bending moment in a beam and who can determine the radius of gyration of a column section may be at a loss to know how to use these calculations, and a knowledge of the methods referred to above will undoubtedly be of assistance to him.

The first step to be taken in determining the steel for any building, is to lay out the framing plan for a typical floor. In order to do this it is necessary to stretch tracing paper over the architect's plan and locate the columns. The architect himself has a great deal to say about the last question. Columns must be placed so as not to interfere with the circulation and not to project into rooms. In loft buildings, department stores, warehouses and buildings used for manufacturing purposes, the location of columns depends more upon engineering than upon architectural requirements. Architects often want to space columns too far apart for engineering requirements and engineers want them too near together to suit the architect. If possible the columns should divide the plan into rectangles or "bays." These bays should have approximate dimensions of eighteen feet by twenty-four feet. Of course there are many cases in which such dimensions are impossible and the rectangles may become larger or smaller, but a span of over twenty-four feet for either beams or girders gives excessive depths and a span of less than eighteen feet often results in uneconomical sections.

Fig. 46 shows a portion of a typical framing plan of a department store. The walls are self-supporting and columns are used only as braces as far as the walls are concerned. The floor load is taken as

200 pounds per square foot. In the bay included between columns 33, 34, 43 and 44 framing for elevator shafts and stair wells is shown. Aside from this framing the plan is perfectly simple.

As shown the girders are shorter than the beams. This is the usual method of framing because it gives beams and girders of com-

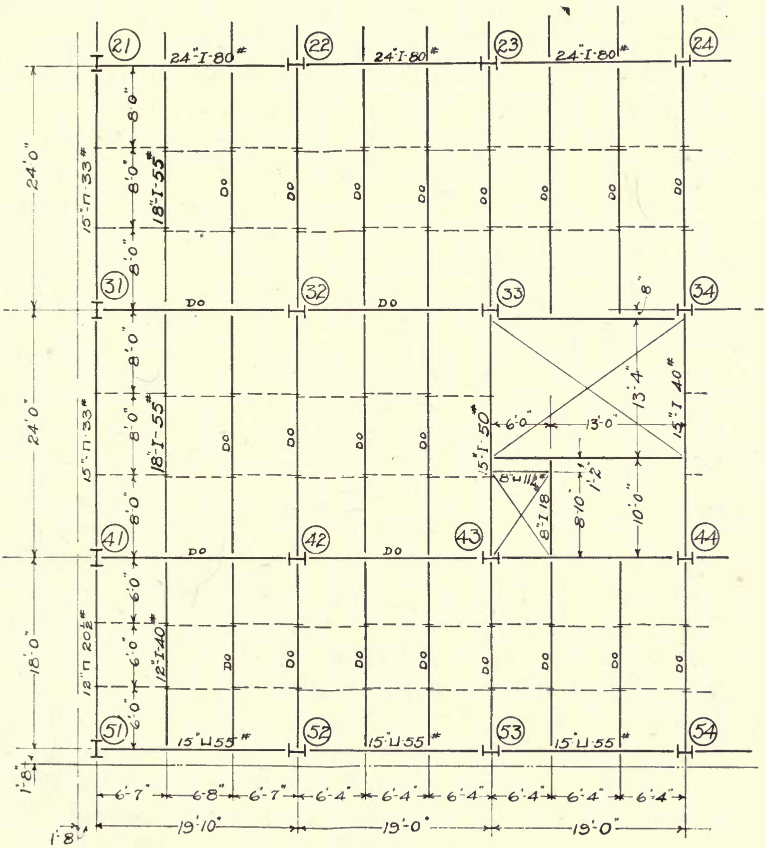


FIGURE 46

paratively the same depth. If the conditions were reversed there would be excessively deep girders and beams that would be — architecturally speaking — "out of scale."

In the original layout the columns are merely indicated by circles and the beams framing directly to the columns are drawn in as shown in Fig. 47. At this point it may be noted that such beams and girders as shown must be framed directly to the column centers in

order to secure stiffness. Occasionally, as between columns 33 and 34, in order to meet architectural requirements it may be necessary to frame a girder a little off center, but this should never be done in such a manner as to make a direct connection between the column and girder impossible.

Once this much is drawn, beams are indicated and a tentative plan is laid out. Then the actual work of figuring is begun.

In order to have a complete set of figures a careful record of all calculations must be kept. Some engineers use books, others have loose leaf binders, and some simply have specially made pads of paper. Whatever the method, every calculation should be recorded and no figures which have not a direct bearing on the engineering problems should be entered in the record.

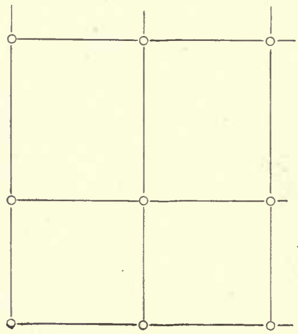


FIGURE 47

To start the calculations, a bay which represents the most common conditions is chosen. Such a bay is found included between columns 21, 22, 31, 32. The engineer starts by making a small sketch of this bay in his notes (Fig. 48). Then, having determined the floor load he finds the total uniform load which a beam has to carry. Each beam is 24 feet long and carries a load that extends over an area 6.6 feet wide by 24 feet long. The total floor load on the beam is then 31,680 pounds. Looking in the handbook, the safe load for an 18-inch, 55-pound I-beam, spanning 24 feet is 39,290 pounds, so this beam will be selected. This completes the calculation for the filling-in beams and a line is drawn. In connection with the above calculation it may be well to call attention to the fact that although a 15-inch 60-pound I-beam will carry the load, the 18-inch beam will be five pounds per foot lighter and this saving for all the beams in all the floor panels will be considerable.

The next consideration is to determine the size of the girder between columns 31 and 32. For the purpose a sketch is made indicating the loads and their positions. The loads are brought to the girder by the beams considered above and act as two concentrated loads. If we consider that all the weight of the floor arches is carried by the beams, there is no uniform load on the girder. As the loading is symmetrical, the reactions will be equal and each reaction will be 31,700 pounds. The maximum bending moment

will be found at the point where either beam frames into the girder. To find I/c simply divide this bending moment in *inch pounds* by S — the safe working stress for steel — or 16,000 pounds per square inch. This is found to be 157 and a 24-inch 80-pound I-beam will be selected. The sizes of these beams and girders must be noted on the plan. The girders and beams in panels 22, 23, 32 and 33 will be the same as those shown above.

In the lower panels it may be well to make a trial to see whether it would be better to frame the beams as shown or to have them parallel to the longest dimension of the bay. First consider the condition indicated on the plan.

A small sketch of the bay should be drawn. The load on the beams will be 23,800 pounds and in the handbook will be found a 12-inch, 40-pound I-beam that will carry the load over a span of 20 feet. A 12″, 20½-lb. channel will be strong enough to carry the load between columns 41 to 51. Girder 41-42 will have two concentrated loads upon it. These loads will be found by adding one-half the load on each beam in panel 21-22-31-32 to one-half the load on each beam in the panel at present under consideration. One-half of 31,680 is 15,840 and one-half of 23,800 is 11,900, so each concentrated load will be 27,740. The section modulus will be 137.2 and a 24-inch, 80-pound I-beam will answer. To find the size of the channel necessary between columns 51 and 52, a sketch should be drawn and the two concentrated loads of 11,900 pounds located. The section modulus is found to be 59 and although I/c for a 15-inch 55-pound channel will be slightly less than this, it will be chosen as the building department will allow a slight difference of five per cent.

The weight of steel per square foot of panel will be determined as follows: One-half of the weight per foot of the girder 41-42 or 40 pounds is assumed to come in the panel. The whole weight of the channel (51-52) or 55 pounds is in the panel. The weight of these is distributed over 18 feet, so to get the weight per square foot add 55 to 40 and divide by 18. $55 + 40 = 95$. $95 \div 18 = 5.3$ pounds per square foot.

Each beam weighs 40 pounds per foot and this is distributed over an area 6.6 feet wide. So each square foot would have $40 \div 6.6 = 6$ pounds of steel from the beams. Add 6 to 5.3 and a total weight of 11.3 pounds per square foot is given as supplied by beams and girders.

In order to test the weight of the panel with the beams framed parallel to the longest dimension, a trial sheet should be started. At the top of this the word "Trial" should be written. (Fig. 49.) A sketch of the panel is made with the beams indicated as shown. The weight upon the beams is then determined as 24,000 pounds and a 15-inch, 42-pound I-beam will be selected. Girder 42-52 will have two concentrated loads of 24,000 pounds each and the section modulus is found to be 108. A 20-inch, 65-pound I-beam will answer the requirements. For the channel between columns 41 and 51 the section modulus of 54 will be obtained — one-half of that of the girder — and a 15-inch, 50-pound channel will be strong enough. A small sketch should be drawn for girder 41-42. This carries a uniform load and two concentrated loads. The total uniform load can be found — from the calculation for beams in panels 41, 42, 51 and 52 — to be 12,000 pounds, and each concentrated load can be determined from the load on the beams in panels 21, 22, 31 and 32. These loads are each 15,840. The maximum bending moment will be found at the center. The upward moment around this point will be caused by the reaction multiplied by one-half the span. The two downward moments are caused by the concentrated load (15,840 lbs.) and one-half the total uniform load.

The weight per square foot of the steel will be found as before. Forty-two pounds of steel in the beam will be distributed over 6 feet, so there will be 7 pounds of steel per square foot from the beam. $50 + 32 = 82$ pounds of channel and girder distributed over 20 feet or 4.1 pounds per square foot. The total weight will be 11.1 per square foot provided we assume that the weight of girder 41-42 will about counteract the light channel 51-52.

The two weights will therefore be 11.1 in the last panel and 11.3 in the former, but there is so little difference in those two weights that it will not be noticeable. The framing shown in Fig. 46 will be adopted because a stiffer frame will result if this method is used.

In Fig. 50 a condition is shown where framing for a stair well occurs and where beam 2-11 carries a brick wall. The loading on the beams in panel 10-11-20-21 is the same as that in panel 41-42-51-52 or 23,800 pounds. Then girder 10-11 carries two concentrated loads of 12,000 pounds approximately and a uniform load caused by the floor. This load equals 10,000 pounds. The maximum bending occurs at the center of the beam and is found to be 105,400 foot-pounds. Now in all previous calculations the usual method of

determining I/c from this has been to multiply by 12 and divide by 16,000. This becomes a somewhat lengthy process and the same result will be accomplished simply by multiplying the load *in thousands of pounds* by $\frac{3}{4}$. The section modulus is then found to be 78.6, and an 18-inch, 55-pound I-beam will be used. The beam used to support the stair construction will carry a uniform load

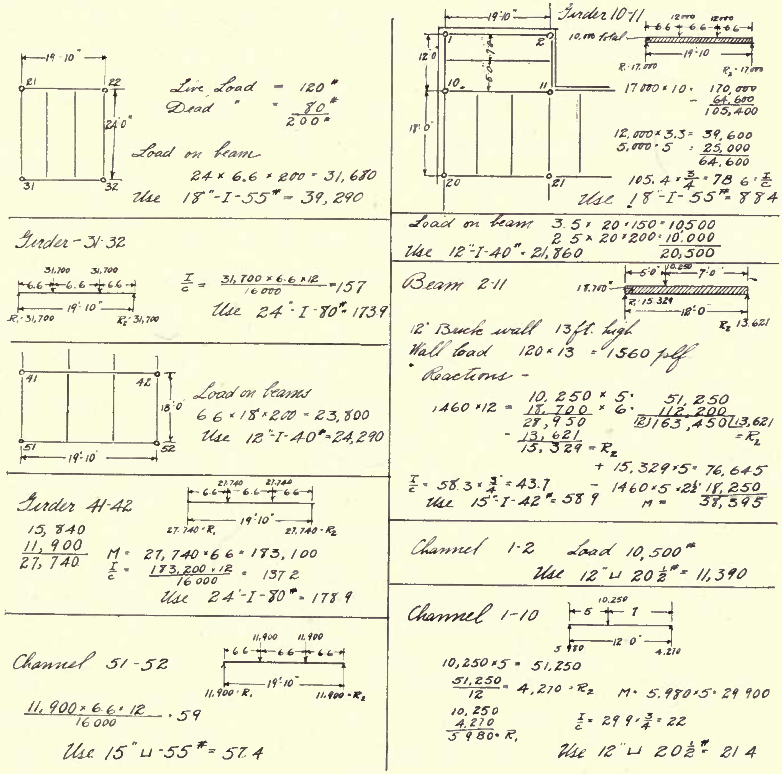


FIGURE 48

from one-half the floor arch and a uniform load from one-half the stair area. The weight per square foot of stair construction is taken as 150 pounds, including both dead and live loads. The total load will be 20,500 pounds and a 12-inch, 40-pound I-beam will support it.

Beam 2-11 carries a uniform load of the wall which is 13 feet high. Engineers always figure brick as weighing 120 pounds per cubic foot so the wall will weigh 1,560 pounds per lineal foot. The

letters p. l. f. stand for the words "per lineal foot" in the calculations. The reactions are found to be 13,621 pounds and 15,329 pounds respectively. The maximum bending occurs where the uniform

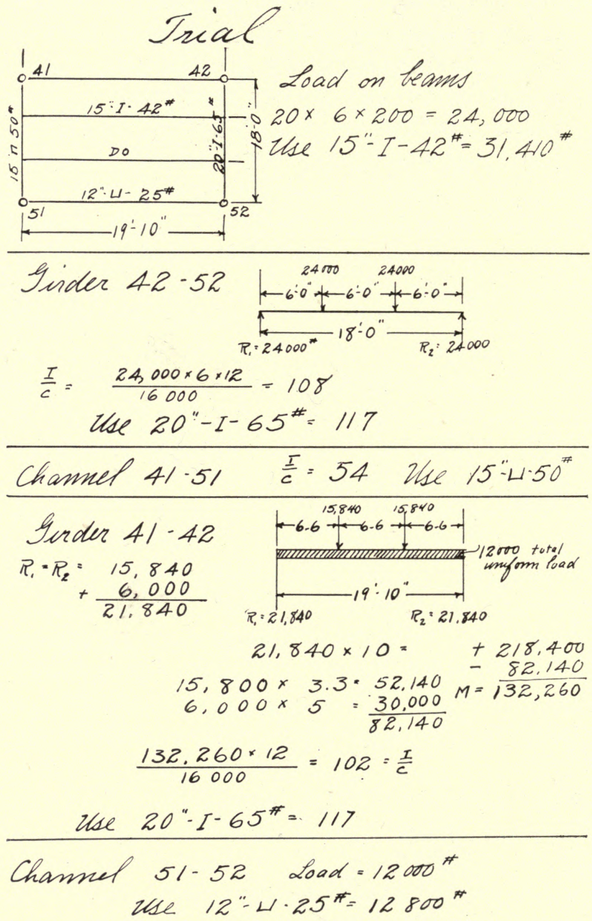


FIGURE 49

load is placed. The architect should check this to see if the shear changes from positive to negative at this point.

The maximum bending moment is found to be 58,395 foot-pounds and the I/c is determined as 43.7. A 15-inch, 42-pound I-beam will be selected. For the channel 1-10 a section modulus of 22 is obtained, the bending being caused simply be the load brought to the

channel by the beam. A 12-inch, $20\frac{1}{2}$-pound channel will be selected, although it is a little light. Channel 1-2 will carry only such a uniform load as is brought to it by the stairs and this will be carried by a 12-inch, $20\frac{1}{2}$-pound channel.

The framing around the elevator shafts and stair well requires but little explanation. Girder 33-34 carries two concentrated loads brought to it by the floor beams. In some cases another concentrated load is considered. This is due to the load caused by the impact of the elevator in case the cables should break and the emergency brake cause the weight of the elevator to come upon the guides. In the present problem this load will not be taken into account as engineers figure that the live load is great enough to take care of any extraordinary conditions such as this. If the architect should adopt the methods referred to in this article he will find that the girder required will have to be an 18-inch, 55-pound I-beam. The stair well load will be considered as a simple uniform load of 150 pounds per square foot coming upon the short beam and upon beam 34-44. The short beam need be only an 8-inch, 18-pound I-beam. The only loads upon the beam framing between beams 33-43 and 34-44 will be the single concentrated load brought by the 8-inch beam and the weight of a 6-inch terra cotta wall. A 15-inch, 25-pound I-beam will be found to be strong enough. A 10-inch, 50-pound I-beam will be framed between columns 33 and 43 and a lighter, 42-pound beam between 34 and 44.

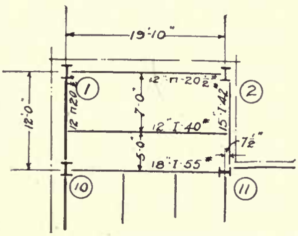


FIGURE 50

The methods given in this article are mere suggestions of those that may be employed in actual practice. The conditions that have been considered are typical ones and although there may be many more complicated ones, the architect need have no fear of them provided he is careful in his diagnosis of the case. The chief difficulties encountered are not due to engineering considerations, but to carelessness in placing the loads or in leaving important loads out of the calculations entirely. The architect is too apt to want to start in figuring before he fully draws out the diagram that represents the methods of loading the beam.

Architects often believe that engineers have a great guessing ability and do not figure out all the conditions. This impression is absolutely wrong. Records of the best engineers in the profession

show that every little detail is gone into, that there is not a single beam in a floor plan that has not been very carefully considered. Those who are most familiar with the situation realize that even with the most careful checking mistakes will occur and, because of this, as little as possible is left to chance.

An architect who found that certain considerations had been overlooked in his plans, excused his oversight to the engineers by saying that he had taken certain things for granted. The answer was made, "Be a good engineer and take nothing for granted."

CHAPTER IX

Column Schedules. Formulas. Loads. Column sections. Checks. Eccentric loads.

THE sets of steel drawings that an architect is usually most familiar with are the framing plans and the column schedule. The method of determining the sizes of beams in a framing plan was taken up — more or less roughly — in the last chapter. This chapter will deal with the considerations involved in the working out of a column schedule. Fig. 51 shows a portion of such a schedule. For the purposes of this chapter all the columns are made of plates and channels, because tables are given for the safe loads to be carried by such columns in both the Cambria and Carnegie handbooks.

In order to understand the methods employed by engineers in developing column schedules, the architect should become thoroughly familiar with the formulas given in Chapter V. The stress per square inch that is allowed by the New York Building Department is given by the formulas $S = 15{,}200 - 58\ l/r$, l being the unsupported length of the column in inches and r the radius of gyration of the column section—also measured in inches. The method of determining r should also be remembered as there often exist cases where ordinary column sections given in the handbook cannot be used.

To these formulas another should be added — one dealing with the conditions of eccentric loading. This is $S = \frac{W_1 + W_2}{A} + \frac{Mc}{I}$. In this formula it is assumed that two kinds of loads exist, one, W_1, coming directly upon the axis of the column and the other, W_2, acting at a distance more than eight inches away from the center line. W_1 is the axial or concentric load and W_2 is the eccentric load. If no attention is to be paid to the bending set up in a column due to the eccentric load, it is plain that S the stress per square inch due to W_1 and W_2 will be $\frac{W_1 + W_2}{A}$ in which formula A is the area of the cross section. This is the first part of the formula given above. There will be, however, extra bending stresses due to the eccentric load — W_2. In Fig. 52 it will be seen that the load W_2 is acting at a distance y from the center of the column.

The moment set up by this load will be Wy and this will be denoted by M.

If the architect remembers the formula for the resisting moment of a beam, $M = S\ I/c$, he can easily see that $S = M \div I/c$. If this S is added to that caused by the concentric load $\frac{W_1 + W_2}{A}$ then the total S in the section will be determined. So the whole formula will be $S = \frac{W_1 + W_2}{A} + \frac{Mc.}{I}$ The use made of this formula will be referred to later.

The first thing to be determined is the load upon the column. In order to do this and also keep a fairly accurate record of all his calculations, the architect should adopt some form of tabulating his loads. The following method of recording loads is submitted, not as an absolute and set form, but as one that has proved fairly satisfactory and one that will furnish the architect with a means of attacking the ordinary problems involved in column design.

Fig. 53 shows a sheet, ruled in such a manner as to allow for the tabulation of the loading upon the columns on each floor. The reason for dividing the live load from the dead loads is that the building department allows a reduction of five per cent. in the live load for each floor below the roof and top floor until fifty per cent. of the load is deducted and after that no further deduction is allowed. A space is left for added loads due to eccentric loading.

To explain the method of proceeding an example will be given. Column 21, as seen in Figure 54, is an interior column having beams and girders framing to it in the ordinary manner. There is no eccentric loading at all. Let it be considered that the live load is 120 pounds per square foot and the dead load 80 pounds per square foot, the total being 200 pounds, 60 per cent. of which is live and 40 per cent. dead load.

The quickest method of finding the load on columns is to consider the area of floor which the column carries. Such an area is included within the lines *ab*, *bc*, *cd*, and *da* (Fig. 54) and is 19 feet wide by 24 feet long. The weight of this is $24 \times 19 \times 200 = 91{,}200$ pounds, 54,720 pounds being live load and 36,480 pounds dead load. A record of these calculations is entered on the ruled sheet, Fig. 53, the live load, dead load, and total all being shown, as coming upon the column at the eighth floor.

The roof load differs from that of the eighth floor, in that the live load in the roof is only 50 pounds as required by the Building Code and the total load will be $80 + 50 = 130$ pounds per square foot. The load upon the column at the roof level will be $24 \times 19 \times 130 = 59{,}300$ pounds — 22,800 pounds live and 36,500 pounds dead load.

The load at the seventh floor would be exactly the same as that on the eighth except that the live load is reduced five per cent. In this connection it may be well to say that the process of dropping off this percentage of the live load at each floor may be made very simple by the use of the slide rule. Dead loads are simply recorded as shown. Ninety-five per cent. of 54,720 is approximately 52,000 pounds. Ninety per cent. is 49,300 pounds and so on. The total load brought to the column by each floor is then found and these totals are added together.

The next step is the selection of the column sections that will withstand these loads. It will be noted on the schedule that the columns are made in sections extending through two floors — that one section extends from the basement to the second floor, and another from the second to the fourth floor. The last section will extend from the sixth floor to the roof and the greatest load upon it is 238,980 pounds. Looking in the table for safe loads for columns it will be found that for an unsupported length of 12 feet a section made of 10-inch, 15-pound channels and 15 by $\frac{3}{8}$-inch plates will support 245,000 pounds. This must be checked to see if it conforms to the Building Code requirements. $r = 4.65$, $l = 12 \times 12 = 144$. $58\,(l \div r) = 58 \times 144 \div 4.65 = 1{,}795$. $15{,}200 - 1{,}795 = 13{,}405 = S$.

The area of the section is 20.17, so the load it will support is $20.17 \times 13{,}405 = 270{,}000$

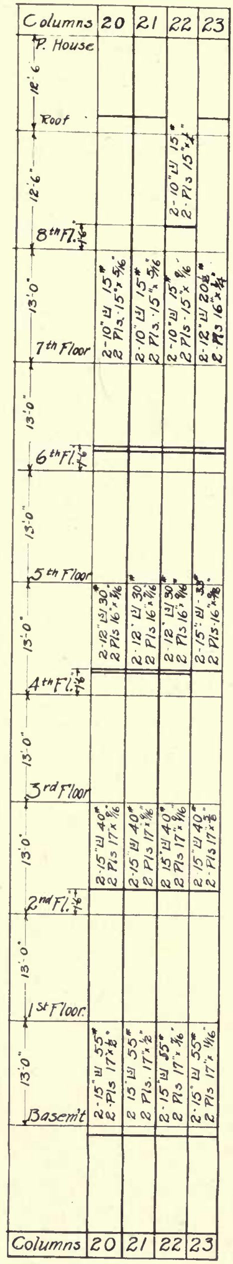


FIGURE 51

pounds. A lighter column might be used so the section will be made of the channels with 5/16-inch plates. It might be well to state that the Carnegie formula for stresses in columns is different from that employed in determining the safe loads for Cambria column sections and both differ from the formula employed by the New York Building Department, but the results are about the same in all cases. The portion of column extending from the seventh floor to the roof is much heavier than necessary, and in some cases engineers take this into account, but the saving resulting from the cutting down of field riveting and splicing and the extra stiffness of the frame makes the use of a long section of column advantageous. The next section extends from the fourth to the sixth floor and the load of 407,840 pounds at the fifth floor determines the design.

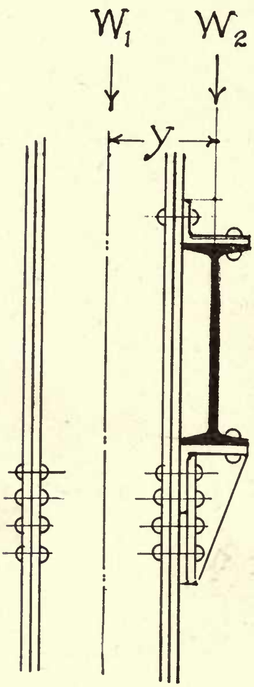


FIGURE 52

The section given in the handbook as being strong enough to withstand this load is made of 12-inch channels weighing 30 pounds per foot, and 16-inch by 7/16-inch plates. The radius of gyration of this section is 5.04 and $58 \times 144 \div 5.04 = 1{,}653$. Subtracting this from 15,200, the stress per square inch is given as 13,547 pounds. The area of the section being 31.64 square inches, the total strength of the section will be $31.64 \times 13{,}547 = 428{,}600$. This section is safe and will be used. To jump from a 10-inch channel section to a 12-inch section requires the use of a butt plate and angles. Such a connection is shown on page 308 in the Cambria handbook, and on page 279 of the Pocket Companion published by the Carnegie Steel Company, and is indicated by the use of double lines on the column schedule.

The section of column 21 extending from the fourth to the second floor must support a load of 565,700 pounds. It will be found that a section made up of 15-inch, 40-pound channels and 17-inch by 9/16-inch plates will be strong enough.

The last section will have to support a load of 712,610 pounds

at the first floor and a section made of 15-inch, 55-pound channels and 17-inch by $\frac{5}{8}$-inch plates will be strong enough.

The next column, number 22, extends to the pent house and carries the load of elevator beams and a portion of the pent house roof. In calculating the sizes of beams necessary to carry the elevator sheave beams the designer had to determine the reactions brought to the columns by these beams. To find the total load upon the column all that is necessary to do is to add the proper reactions together. For the design of columns alone it is absolutely necessary for the engineer or architect to keep complete records of all the calculations for beams in the framing plans.

There is no separation of live and dead loads for the roof and pent house beams as there is no percentage of the live load deducted until the eighth floor is reached. It will be noticed on the column schedule that a section extends from the sixth to the eighth floor and another from the eighth to the pent house roof. Both for the economy of material in the column section and for the requirements of shipping this is necessary. The load that the top section must stand is found at the roof. This is 206,435 pounds, and a section made of two 10-inch, 15-pound channels and two 15-inch by $\frac{1}{4}$-inch plates will answer. The rest of the calculations are the same as for column 21.

So far all loads have been concentric or axial loads, but column 23 has to support a wall load which sets up bending stresses in the column. The method of finding column sections of sufficient strength to withstand eccentric loads is not direct, but involves assuming sizes and then checking. Of the 55,970 pounds brought to column 23, 25,500 pounds are eccentric as they are due to wall loads. The beams that carry the wall are 8 inches off the center. (Fig. 54.) The moment, M, set up by the eccentric load is $25{,}500 \times 8 = 204{,}000$ inch-pounds.

Referring to the formula in the first part of this article, it will be found that the added stress per square inch due to the eccentric load is $M \div I/c$ or Mc/I. I/c for a column section such as those used in this article can be found in the handbook under the heading "Moments of Inertia and Section Moduli for Plate and Channel Columns." Assuming, for reasons that will be given later, that a section made of two 12-inch, 20.5-pound channels and two plates 16 inch by 7/16 inch will be strong enough, I/c can be found to be 123.9. $M \div I/c$ will be $20{,}400 \div 123.9 = 1{,}647$. This is the stress

		Col. #20		Col. #21		Col #22		Col #23	
Pent House	Dead	Same as 21							
	Live								
	Ecc.								
	Total					105,600			
	Section								
Roof	Dead			36,500					
	Live			22,800				55,970	
	Ecc.								42,900
	Total			59,300		100,835	206,435		
	Sect'n.					2-10" ⊔ 15#	2 pls 15"x1/4"		
8th Floor	Dead			36,480		25,600		43,703	
	Live			54,720		38,400		27,895	
	Ecc.								59,000
	Total			91,200	150,500	64,000	270,435	71,600	127,570
	Sect'n.								
7th Floor	Dead			36,480		25,600		43,705	
	Live			52,000		36,480		26,400	
	Ecc.								59,000
	Total			88,480	238,980	62,080	332,515	70,105	197,675
	Sect'n.			2-10" ⊔ 15#	2 pls 15"x5/16	2-10" ⊔ 15#	2 pls 15"x9/16	2-12" ⊔ 20#	2 pls 16"x1/4
6th Floor	Dead			36,480		25,600		43,705	
	Live			49,300		34,600		25,060	
	Ecc.								52,000
	Total			85,780	324,760	60,200	392,715	68,765	266,440
	Sect'n.								
5th Floor	Dead			36,480		25,600		43,705	
	Live			46,600		32,600		23,700	
	Ecc.								52,000
	Total			83,080	407,840	58,200	450,915	67,405	333,845
	Sect'n.			2-12" ⊔ 30#	2 pls 16"x7/16	2-12" ⊔ 30#	2 pls 16"x9/16	2-15" ⊔ 33#	2 pls 17"x3/8"
4th Floor	Dead			36,480		25,600		43,705	
	Live			43,800		30,700		22,300	
	Ecc.								52,000
	Total			80,280	488,120	56,300	507,215	66,005	399,850
	Sect'n.								
3rd Floor	Dead			36,480		25,600		43,705	
	Live			41,100		28,800		20,900	
	Ecc.								52,000
	Total			77,580	565,700	54,400	561,615	64,605	464,455
	Sect'n.			2-15" ⊔ 40#	2 pls 17"x9/16	2-15" ⊔ 40	2 pls 17"x9/16	2-15" ⊔ 40#	2 pls 17"x7/16
2nd Floor	Dead			36,480		25,600		43,705	
	Live			38,350		26,850		19,500	
	Ecc.								52,000
	Total			74,830	640,530	52,450	614,065	63,205	527,660
	Sect'n.								
1st Floor	Dead	Same as 21		36,480		25,600		43,705	
	Live			35,600		24,950		18,100	
	Ecc.								52,000
	Total			72,080	712,610	51,550	665,615	62,805	589,460
	Sect'n.			2-15" ⊔ 55#	2 pls 17"x1/2	2-15" ⊔ 55#	2 pls 17"x1/2	2-15" ⊔ 55#	2 pls 17"x1/2

Figure 53

per square inch set up by the eccentric load. To find the load that would cause an equal stress per square inch if exerted over the entire area of the column, simply multiply this unit stress by the area of the section. $1{,}647 \times 26.06 = 42{,}900$ pounds. This load is placed in the outside column in Fig. 53 and is not added until the area of the section of the column is to be determined.

The same process is gone through to find the loads for the eighth floor. The actual eccentric load being 35,000 pounds, it will be found that a concentric load of 59,000 pounds will have the same effect as the moment set up by the eccentricity. It will be noted that there is no separation of the live and dead loads at the roof level, but that this must be done at the eighth floor. The same

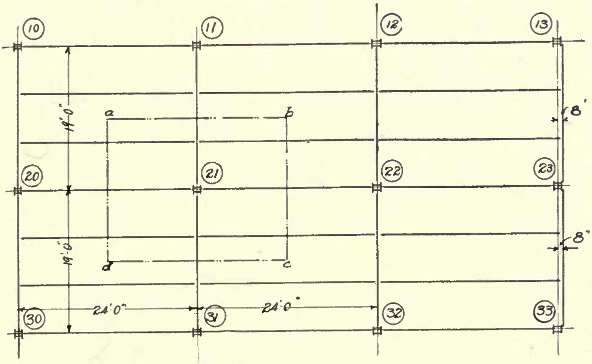


FIGURE 54

process is gone through for the seventh floor, but the live load is reduced 5 per cent.

The section of column extending from the sixth floor to the roof will have to be designed to carry the load of 256,675 pounds — the sum of the load brought to the column by the roof, the eighth, and seventh floors plus the greatest load due to eccentricity. In the table of safe loads a column made of 12-inch, $20\frac{1}{2}$-pound channels and 16-inch by $\frac{1}{4}$-inch plates will give about the proper strength. The radius of gyration is 5.23. $15{,}200 - 58 \times \frac{144}{5.23} = 15{,}200 - 1{,}600 = 13{,}600$ pounds per square inch. Multiplying this by the area and the result — 272,000 pounds — will be near enough.

The next section will only have to be made strong enough to withstand the concentric load of 197,675 pounds brought down by the upper section, plus the floor loads at the sixth and fifth floors

and the eccentric load at one floor. The eccentric loads on the top section will not be added to those of the lower portion. Thus it may be assumed that there will be an added concentric load of about 59,000 pounds approximately. So at the fifth floor level there will be an *assumed* load of 197,675 + 140,000 + 59,000 = 396,675.

For this load it will be better to choose a section made of 15-inch channels, as the weight of a light 15-inch channel section will be about the same as a fairly heavy 12-inch channel column, and the section modulus will be much larger, giving more resistance to bending. The section modulus of a column made of 15-inch, 33-pound channels and 17-inch by $\frac{3}{8}$-inch plates is 175.1 and the area 32.55 square inches. The added eccentric load will be 52,000 pounds. Adding the loads, as shown in Fig. 53, the load at the fifth floor will be 333,845 + 52,000 = 385,845 pounds. Checking the section as before it will be found to be strong enough to withstand this load.

Applying the principles outlined in this chapter to the remaining sections the results will be found to correspond to the figures in the column schedule.

CHAPTER X

Graphical Methods of Indicating Forces. Composition of forces. Resultants and equilibrants. Points of application. Graphical methods applied to simple beams.

A FORCE is completely defined when its magnitude, direction, and point of application are known. In Fig. 55 is a simple beam with a concentrated load upon it. The magnitude of the load is given by the figures, — 2,000 pounds, — the direction is indicated by the arrows, — downward — and the location by the dimensions. In this manner the load is entirely defined. There is, however, another method of showing the three elements of a force and that is by means of a straight line. In Fig. 55*a* the line is shown divided into equal parts. Each part is supposed to represent 1,000 pounds; the total length of the line represents 2,000 pounds. In this manner the length of the line represents the *magnitude* of the force. The arrow shows the direction, and the point *p* may give the point of application.

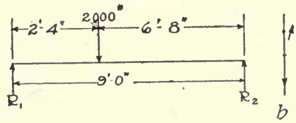


FIGURE 55

As lines may represent forces, it is possible to use them to determine the action of forces. Two forces acting at a point may be replaced by a single force acting at that point. In Fig. 56 two forces, P_1 and P_2, act at the point *p*. The same effect will be produced if a single load — R — acts at that point. The method of determining R is by means of the "parallelogram of forces." It will be noticed that R is the diagonal of a parallelogram the sides of which are parallel to P_1 and P_2 respectively. Instead of constructing a parallelogram a triangle may serve the same purpose (Fig. 56*a*) in which P_1 and P_2 form two sides and R the third. R is always used to designate the single force which will take the place of a system of several forces and is called the *resultant*. To find the magnitude and direction of the resultant of two forces, all that is necessary to do is to construct a triangle, as shown in Fig. 56*a*, with the two known

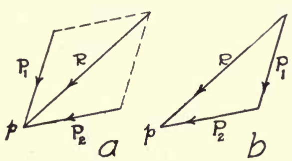


FIGURE 56

forces represented parallel to their original direction and the arrow head of one placed at the butt of the other.

Fig. 57 represents another condition. In this case it is necessary to find not the resultant, but a force that will oppose the two known forces and set up a condition of equilibrium. E is found in the same manner as R, but it will be seen that the arrow points in an opposite direction. This force, if acting at p at the same time as P_1 and P_2, will force p to remain exactly as it is. E then sets up a condition of equilibrium and is known as the *equilibrant.* The equilibrant is equal in magnitude to the resultant, but acts in the opposite direction. In Fig. 57*b* the forces are shown in the same manner as in 56*b*, but it will be noticed that all the arrows tend to follow *around* the triangle. When this is the case there is always a condition of static equilibrium.

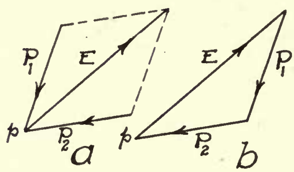


FIGURE 57

So far there have only been two known forces, but it is possible to determine the magnitude and direction of the resultant or equilibrant of any number of forces provided the magnitude and direction of each one of them is known. In Fig. 58 are shown four forces P_1, P_2, P_3, and P_4 acting in the directions indicated.

P_1 and P_2 can be drawn as shown and resolved into the resultant R_1. A second triangle made of R_1 and P_3 and R_2 can be formed,

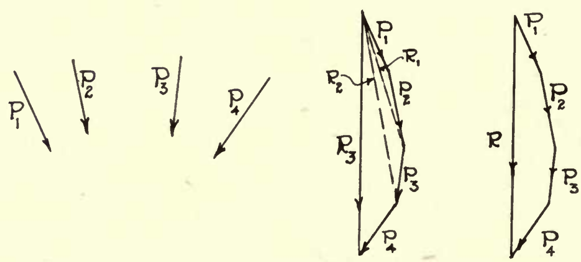


FIGURE 58

R_2 being the resultant of R_1 and P_3. A third triangle made of R_2, P_4, and R_3 will give the resultant of the two forces R_2 and P_4. From this it can be seen that R_3 is the resultant of all four forces, P_1, P_2, P_3 and P_4. Instead of drawing R_1 and R_2 in order to find the resultant of a system of forces, all that is usually drawn is the final resultant as shown in the third diagram of Fig. 58. It will be noticed that all

the arrows except that of R follow around the diagrams. To find the equilibrant of three forces the same method is employed as shown in Fig. 59. Note the direction of the arrows.

Although this method gives the magnitude and direction of either R or E the points at which such forces are to be applied — so as to give the same results as the system of forces or to put the system into a condition of equilibrium — has not yet been determined. A further elaboration of the figure will give the required points.

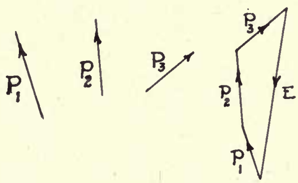


FIGURE 59

In Fig. 56 it was shown that two forces could be resolved into a single force. By reversing the process it can be shown that a single force may be divided into two forces. P_1 in Fig. 60b acts through the point p. The magnitude of P_1 is shown by the line AB (Fig. 60a). To get the magnitude and direction of any two forces that will give the same result as P_1, take any point O, and draw OA and OB. Also draw $O'A'$ and $O'B'$ in Fig. 60b parallel to OA and OB and passing through p. These two forces will produce the same effect on p as the single force P_1. To show that O can be taken at any point, note the effect in Fig. 60 (c and d).

Fig. 61 shows a system of four forces divided up in this manner. OA, OB, OC, OD and OE are the components of P_1, P_2, P_3 and P_4, respectively. These can be transferred to A as shown starting with any point p on P_1 and drawing $O'A'$, $O'B'$, $O'C'$, etc., parallel to OA, OB, OC, etc.

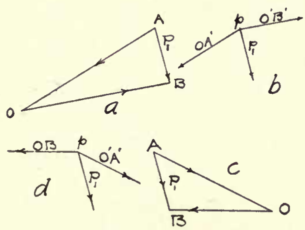


FIGURE 60

It will be noticed that OA and OE act as components of R in the same manner as OA and OB act as components of P_1. At the intersection of $O'A'$ and $O'B'$, (Fig. 61a) a point is found through which R must act. In this manner the point of application of R can be determined, and all that it is necessary to know about the resultant is known — the magnitude and direction being given in b.

At this point it is suggested that the architect draw several diagrams of his own, selecting any number of forces of any magnitude and find the resultants and points of application.

By using the same method, it is possible to find the direction and magnitude of two unknown parallel forces, provided we know their points of application. Given the forces P_1, P_2, P_3 and P_4, as shown in Fig. 62, and the two points X and Y, through which it is desired to pass two forces, parallel to each other, that it will set up a condition of equilibrium. In b the diagram — known as the force polygon — is drawn, in which P_1, P_2, P_3 and P_4 are carefully laid out equal in magnitude (length) and parallel in direction to the respective forces in a. F and A are connected and the *direction* of the two parallel forces is determined. Through X and Y draw lines parallel to FA. It is now necessary to find the magnitude of these two forces. Locate O at any point and draw OA, OB, etc. Through *any* point on P_1, draw $O'A'$ and $O'B'$ and complete the diagram. It will be noticed that continuing $O'A'$ until it intersects E_1, and also continuing $O'F'$ until it intersects E_2, points on E_1 and E_2 are found that must be connected by $O'J'$ to close the diagram. By drawing OJ parallel to $O'J'$ from O to a point on FA, FA is divided into two parts. FJ gives the magnitude of E_2 and JA gives the magnitude of E_1. In this manner it is possible to determine the magnitude of two parallel forces, acting through any two points that will set a system of forces in equilibrium.

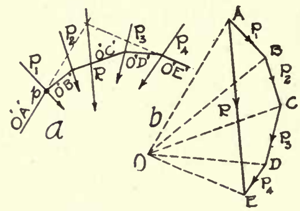


FIGURE 61

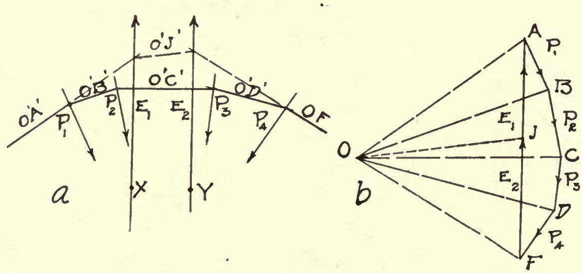


FIGURE 62

Now the obvious question is, what is the use of all this? Perhaps the answer is best found in a practical problem. Given a beam 18 feet long and carrying loads shown in Fig, 63, it is necessary to find the reactions. Lay off the polygon as indicated in (b) with AB parallel in direction and equal in magnitude to the 120 pound load,

BC, and *CD* bearing the same relations to the 580 and 600 pound loads, respectively, and the sum — *AD* — representing the total load upon the beam.

Complete the diagram in (*a*) and it will be found that *O'A'* intersects R_1 and *O'D'* intersects R_2 and that these intersections may be connected by *O'F'*. Drawing *OF* in (*b*) parallel to *O'F'*, *DA* is divided into two forces, *DF* which is equal to R_2, and *FA'*, which is equal to R_1. Scaling off these lengths, it can be found that R_1 equals 380 pounds, and R_2 equals 920 pounds. These results can be checked as follows:

	120 pounds ×	5 feet =	600 foot-pounds
	580 pounds ×	12 feet =	6,960 foot-pounds
	600 pounds ×	15 feet =	9,000 foot-pounds
Totals	1,300 pounds		16,560 foot-pounds

$16{,}560 \div 18 = 920$

$1{,}300 - 920 = 380$

By combining (*a*) and (*b*) in diagram (*c*) the shear diagram is easily drawn. The base line is drawn in line with *F* (*b*). The shear at R_1 is drawn parallel and equal to *FA*. The loads "step" off as shown, all points being transferred from either *a* or *b*. It will be found that this is a graphical method of showing that the sum of the reactions must equal the sum of the loads. Another thing worthy of notice is that the forces *AB*, *BC* and *CD* are read *downward* and *DF* and *FA* are read *upward*, and that both diagrams are closed. This means that R_1 and R_2 are upward forces and that for conditions of equilibrium the diagrams must close. The point of no shear is found under the 580-pound load and this might be told from the fact that *F* falls between *B* and *C*.

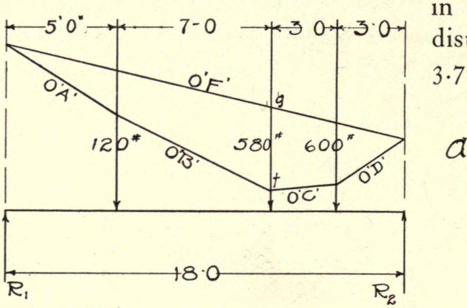
So far we have been able to find the reactions and the shear diagram by means of the graphical methods. It can be shown that the bending moment at any point in the beam can also be determined. Let it be necessary to find the maximum bending moment. This occurs under the 580-pound load.

Two new lines must be drawn, a perpendicular is dropped from *O* to *AD* and the length of this line is designated as *H*. This is known as the *pole distance*, and *O* is known as the *pole*. In other words the perpendicular distance from the pole to the resultant is the pole distance. This is measured in the same units as *AB* or *BC*, or

force units. As O can be selected at any point, the distance H can be taken as any distance, but for convenience it is usually given some even value. In this case it is equal to a distance that would scale 1,000 pounds.

The other line is one parallel to AD and passing through the 580-pound load, cutting the diagram at f and g. As all distances in (a) are measured in feet the distance fg is found to scale 3.72 feet.

To find M under the 580 pounds all that it is necessary to do is to multiply H (in pounds) by fg (in feet) and the product, 3,720 foot-pounds, will be the maximum bending moment. In this manner, all the information necessary for the design of a beam can be found by graphical methods. Checking the value for M:



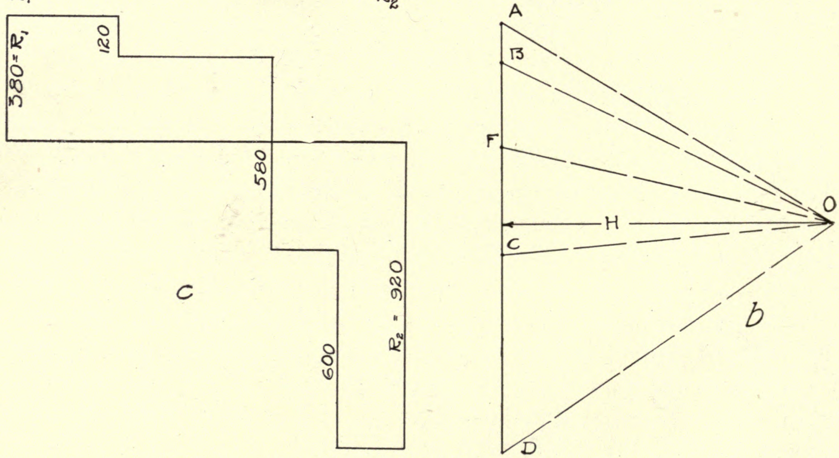


FIGURE 63

$$+\ 380 \text{ pounds} \times 12 \text{ feet} = 4{,}560 \text{ foot-pounds}$$
$$-\ 120 \text{ pounds} \times 7 \text{ feet} = \underline{840} \text{ foot-pounds}$$
$$M = 3{,}720 \text{ foot-pounds}$$

When a uniformly distributed load occurs, a more or less approximate method must be employed. The uniform load is divided into

a number of small concentrated loads. Assuming that there is a uniformly distributed load of 100 pounds per foot over a beam 16 feet in length, it is necessary to divide this load into smaller units — 100 pounds — and treat the problem as though there were sixteen concentrated loads, as shown in Fig. 64. The diagrams are drawn

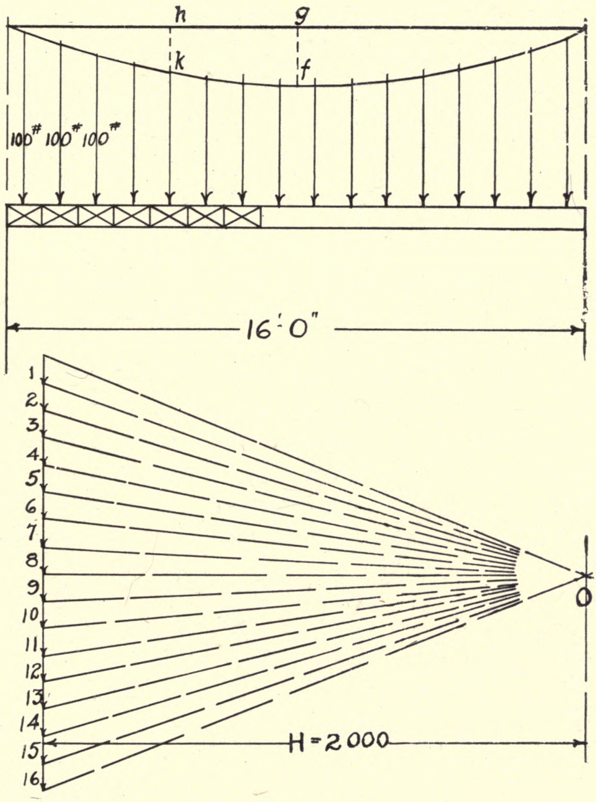


FIGURE 64

as shown, the pole distance being taken as 2,000 pounds. *gf* in this case measures 1.6 feet and the maximum bending moment will be $1.6 \times 2{,}000 = 3{,}200$ foot-pounds. This can be checked by the formula $M = \frac{1}{8}Wl$. $\frac{1}{8} \times 1{,}600 \times 16 = 3{,}200$ foot-pounds.

It must be remembered that not only is it possible to discover the maximum bending moment, but any moment can be found at any point in the beam by this method. At a distance 4-feet 6 inches from R_1 the distance (*hj*) scaled on the diagram is 1.29 feet,

so the bending moment at this point is 2,580 foot-pounds. To check this:

$$\begin{aligned} +\ 800 \text{ pounds} \times 4.5 \text{ feet} &= 3{,}600 \text{ foot-pounds} \\ -\ 450 \text{ pounds} \times 2.25 \text{ feet} &= \underline{1{,}012} \text{ foot-pounds} \\ M &= 2{,}588 \text{ foot-pounds} \end{aligned}$$

These problems have been given in order to show how the action of forces can be determined, and to familiarize the architect with graphical methods. These cases are seldom used in practice, but in some instances are used as checks to locate a possible error in calculation. For this purpose the graphical determination of moments is very valuable.

The principal use of such methods is, however, in determining the stresses in roof trusses. Trusses, having loads due to the roof construction, have to be designed to withstand the vertical pressure of such loads. There are also wind loads that are considered as coming upon the roof in a direction perpendicular to the upper chord of the truss. This makes the use of such methods as have been outlined in this chapter of great value, as they can be used to determine the action of forces in any direction or of any magnitude.

In the next chapter the design of roof trusses will be taken up, but to understand the considerations involved it will be necessary to understand at least the most important of the methods given above.

CHAPTER XI

Simple Truss Problems. Fan truss. Fink truss. French truss. Wind load diagram, both ends of truss fixed.

IN the last chapter the methods of determining the action of forces by means of graphical methods was considered. The most practical use to which these methods are put is that of designing roof trusses. The average architect regards the design of trusses as a difficult matter, but there is no need of this.

A truss may be represented by a simple triangle as shown in Fig. 65 by the lines *XY*, *YZ* and *ZX*. Suppose a force of 100

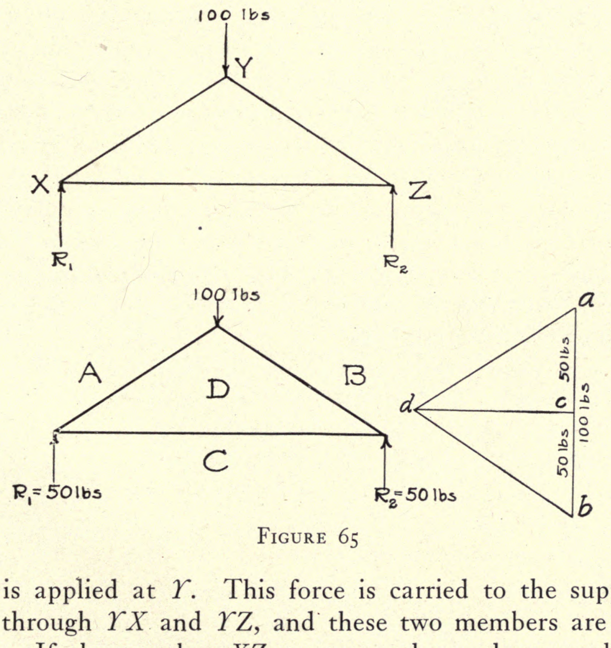


FIGURE 65

pounds is applied at *Y*. This force is carried to the supports R_1 and R_2 through *YX* and *YZ*, and these two members are in compression. If the member *XZ* were not there, there would be an outward thrust at each reaction. *XZ* acts as a tie, holding the ends of the truss in position. This member is in tension.

The problem that a designer has to decide is the exact amount of compression or tension that occurs in any particular member.

In order to do this a system of lettering is employed known as "Bow's Notation."

Note the positions of *A*, *B*, *C* and *D* in Fig. 65*a*. The line *XY* now is between *A* and *D*. The line *YZ* is between *B* and *D*. The 100-pound load falls between *A* and *B*. This means that instead of using the letters at the ends of a line to designate the line, the letters on either side of it are used. In other words *XY* becomes *DA*, the force of 100 pounds becomes *AB*, *YZ* becomes *BD*, and *XZ* is known as *CD*. In Fig. 65*a* it will be seen that *X*, *Y* and *Z* are done away with altogether. The reactions also are lettered in this manner. R_1 is *CA* and R_2 is *BC*. The diagram in which these letters are shown is known as the *truss diagram*. To the right of the truss diagram, Fig. 65, is another known as the *stress diagram*.

When the stress diagram is drawn the use of Bow's Notation becomes apparent. The three known forces are the two reactions and the downward load. On the stress diagram lay off *ab* parallel to *AB* and equal to 100 pounds. In order to find the stresses in *DA* and *BD* draw a line through *a* parallel to *AD*, and one through *b* parallel to *BD*, and the point of intersection of these lines must be *d*. By measuring *da* or *bd* the magnitude of the stresses in the compression members can be determined. The amount of tension in the lower member can be found by drawing a line through *d* parallel to *DC* and a line through *a* which is parallel to *CA*. This last line will coincide with the line *ab*. The point of intersection of the vertical with the horizontal line must be *c*. By measuring *cd* the stress in *CD* is determined.

Once *c* is established the amount of weight coming upon the supports is known. It has already been pointed out that *CA* and R_1 are the same, and this is true of *BC* and R_2. *ca* and *bc* are both given in the stress diagram. It will be noted that *bc* equals *ca* and that each equals one-half of *ab*. As the load *AB* is placed directly in the center of the span, it is plain that each reaction must equal one-half the load.

In all the work in which graphical methods are employed the lines that give the magnitude of stresses, in the stress diagram, must be parallel to the members in which the stresses exist in the truss diagram. In other words, *ab* is parallel to *AB*, *bc* to *BC*, and *cd* to *CD*.

All trusses are not as easily developed as the one given above. The principles of determining the stresses in the members are, however, exactly the same in all. The truss shown in Fig. 66 is known

as a "Fan Truss" and can be used to span over openings of from 20 to 35 feet. The points numbered 1, 2, 3, 4 and 5 are known as *panel points*, and the load upon the truss is generally considered as acting as concentrated loads at these points. For purposes of demonstration let it be assumed that a force of 100 pounds acts at 1, a force of 200 pounds at 2, another at 3, and so on as shown in the figure. The truss diagram is lettered as shown according to Bow's Notation.

The next step is to lay off the stress diagram, starting with the forces already known. These forces are *BC*, *CD*, *DD'*, etc. The reactions are also known and so the points *a* can be established, *ab* being one-half the length of *bb'*.

The unknown forces are found by considering each joint separately. Start at the first panel point and read the forces in a *clockwise* direction.

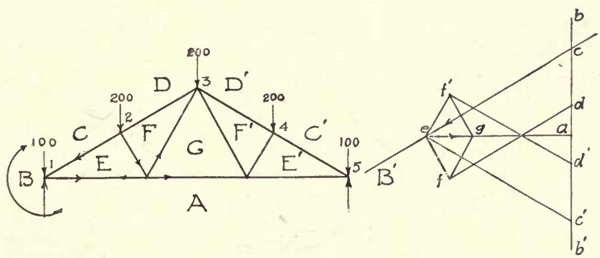


FIGURE 66

This means that the order in which the forces are read is that indicated by the arrow (Fig. 66) and corresponds to the direction taken by the hands of a clock. In other words the forces acting at the first panel point are read as follows: *AB*, *BC*, *CE*, *EA*. Looking at the stress diagram, *ab* and *bc* are known *ce* is not known but its direction is parallel to the upper chord of the truss and the point *c* is known. Draw a line through *c* parallel to *CE* and continue it indefinitely. Neither is *ea* known, but *a* is established, and it is obvious that if *ea* is parallel to *EA* it is a horizontal line, so by producing a horizontal line through *a* until it intersects the line through *c*, the point *e* is found. *e* is common to both *ce* and *ea* and therefore, the intersection of the two lines gives the point that was to be found. By scaling the length of *ce* and *ea* the magnitude of the stresses in the corresponding members of the truss are found.

In order to tell whether these stresses are compressive or tensile forces a very simple process is employed. As the forces are laid off on the stress diagram, *ab* is read up. This stands to reason as

AB is the left reaction — R_1 — and therefore acts up. *bc* is *down.* *ce* acts down and to the left. If an arrow is placed on *CE* indicating the direction in which *ce* is read, this arrow must point toward the panel point around which the forces are taken. This means that *CE* is in compression. When the direction is *toward* the joint the stress is compressive.

ea is read from left to right. An arrow head indicating this direction on *EA* is away from the panel point. This means that

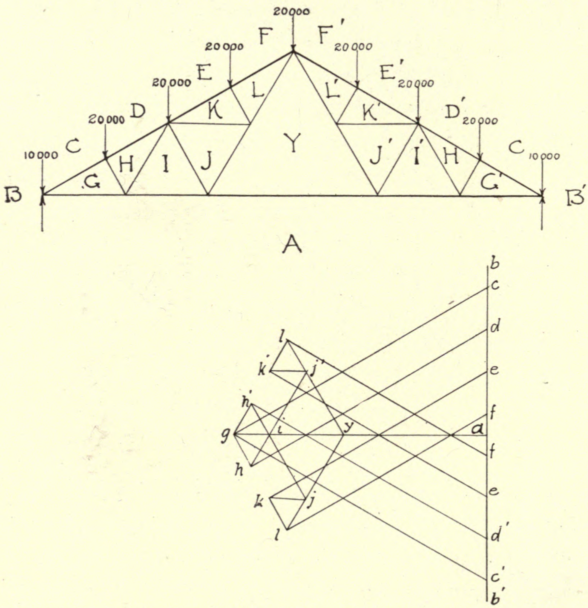


FIGURE 67

EA is in tension. When the direction is *away* from the joint the stress is a tensile stress. The fact that *CE* is in compression and *EA* is in tension is apparent even without this demonstration.

Next consider the stresses and the force acting at the second panel point. *ec* is known. *cd* is also laid off on the stress diagram. The only things known about *df* are the point *d* and the direction. Draw through *d* a line parallel to *DF*. The next member to be found is *fe*. The point *e* is established and through it draw a line parallel to *FE*. The intersection of the two last lines gives the point *f*.

The next two stresses to be found are those in *FG* and *GA*. In order to do this consider the stresses around the joint in the lower

member. *ae* is known, and so is *ef*, but only the directions of *fg* and *ga* are given. Through *f* and *a* draw lines parallel to *FG* and *GA*, respectively, and the intersection establishes the point *g*.

By the methods already outlined the stresses in all the other members of the truss can be determined. The complete diagram is shown in Fig. 66.

To find whether *EF* and *FG* are in tension or compression it will be necessary to follow around the stress diagram in the same manner as before. Reading around the joint in the lower chord, *ae* is read *away* from the joint and is a tensile force; *ef* is toward the joint and is, therefore, a compressive force; *fg* and *ga* are both read away from the joint and so both *FG* and *GA* are subject to pulls of the magnitudes given in the stress diagram.

A diagrammatic representation of a Fink truss is shown in Fig. 67 and as this type of truss is somewhat different from many others it will be considered next. All the loads are considered as acting at the panel points as in the truss already considered. The downward loads are *BC*, *CD*, *DE*, *EF*, *FF'*, etc. The upper reactions are *BA* and *AB*. As the truss is symmetrically loaded the reactions will be equal and each will equal one-half the total load. The point *a* on the stress diagram falls half way between *b* and *b'*, and all the other loads are laid off in exactly the same manner as in the other examples.

The next step is to determine the stresses in the members at the first panel point. *ab* is known and so is *bc*. *cg* and *ga* can be found in the same manner as *ce* and *ea* were found in Fig. 66. *gc*, *cd*, *dh* and *hg* are also easily found. The next joint is in the lower chord and *ag*, *gh*, *hi* and *ia* offer no particular difficulties. At panel point No. 3, however, difficulties arise. There are three unknowns, *ek*, *kj*, and *ji*, and without special information it will be impossible to find the points *k* and *j* on the stress diagram. If it were possible to find either one of these two points, the other can be found. Now it is a characteristic of the Fink truss that the points corresponding to *g*, *h*, *k* and *l* are all in line with each other as shown in the stress diagram, Fig. 67, and so, by continuing *gh* until it intersects the line through *e*, the point *k* is found. This method can be proved to be correct, but the proof would involve the architect in calculations more or less complex, and which for all practical purposes are useless.

Once *k* is established, the point *j* is found by drawing a line through *i* parallel to *ij*, and another through *k* parallel to *kj*. The

intersection of these two lines gives the desired point. It is then possible to read the stresses in *ih*, *hd*, *de*, *ek*, *kj* and *ji*. The whole process depends upon finding the point *k*.

All other stresses can be found without any particular difficulty. This is not only true of this particular truss but of all trusses. The methods given above are those used in designing the most complicated arched trusses or those having unsymmetrical loads. A condition in which there are loads on the lower member is shown in Fig.

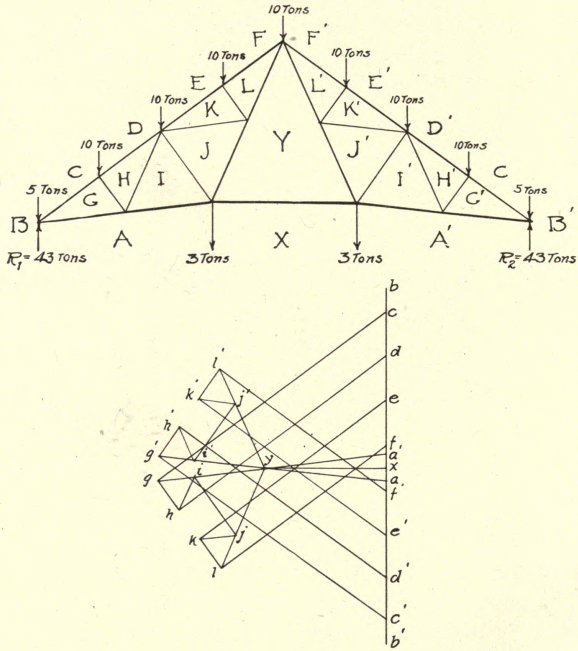


Figure 68

68, and the only new problem is the method of laying off these loads. It will also be noted that the members *AG* and *AI* are not horizontal. This form of Fink truss is sometimes given the name of French truss, and is useful when headroom is required.

The loads are laid off, to begin with, in much the same manner as before, until the point *b'* is reached. The next force, *b'a'*, is the reaction R_2. In all previous examples this has been laid off on the stress diagram equal to one-half the distance *bb'*. In this case *b'a'* is equal to *more* than one-half *bb'*. As the loads are symmetrical,

R_2 is equal to one-half the *total* load or 43 tons. Note that the 3 tons on the lower chord is figured in this reaction. The point a' falls just under f. $a'x$ is the next force, and xa and ab bring one back to the starting point b again.

Once these forces are laid off in order and according to a very logical arrangement, the work of finding the stresses in the members of the truss is undertaken. ab, bc and cd are laid off as usual but it will be noted that ga is *not* horizontal. The only horizontal line in the stress diagram is yx and YX is the only horizontal line in the truss diagram. In the diagrams for both Fink trusses it is well to note that l and j and y are all in line. This relation serves as a check.

The stress diagrams that have been given so far have been developed to give the stresses in all the members of the truss. There is no need of this in actual practice, and usually only one-half the diagram is shown. It will be noted that $a'g'$ is the same length as ag, and the same relation holds good throughout the entire diagram.

All the forces that have been mentioned so far have been downward loads. In case wind loads are to be considered a new element enters into the design. The usual method is to determine the stresses due to the vertical forces and then draw a second stress diagram in which the wind loads only are taken into account.

These wind loads are considered as acting at the panel points in exactly the same manner as the dead loads. The method of determining them will be considered later. The direction of these loads is always taken as perpendicular to the upper chord of the truss. This condition is shown in Fig. 69. The determination of the stresses in the members depends upon the manner in which the ends of the truss are held. In mill construction both ends of a truss are fixed and this is true of many trusses that are used in building construction. When one end of the truss is anchored to a masonry wall the other end may rest upon an iron plate to allow for expansion and contraction. When a truss is over 70 feet in length the free end is often placed upon rollers. The need of this is apparent if one considers the difference in temperature between summer and winter and the expansion that may take place in 70 feet of steel.

When both ends are fixed the reactions act in a direction opposite to the wind loads, that is, perpendicular to the upper chord of the truss. The stress diagram is laid off in much the same manner as those already considered except the loads are not vertical. There

is only need for one diagram, for all forces and stresses are the same whether the wind blows from the right or left.

The important difference between this condition and those given above is that it is necesary to determine the reactions. In the case of a simple beam the method of finding R_2 was to determine the total downward moment around R_1 caused by the loads, and to divide this by the span. Exactly the same process is employed in reference to the truss. The total wind pressure is 4,000 pounds which may be

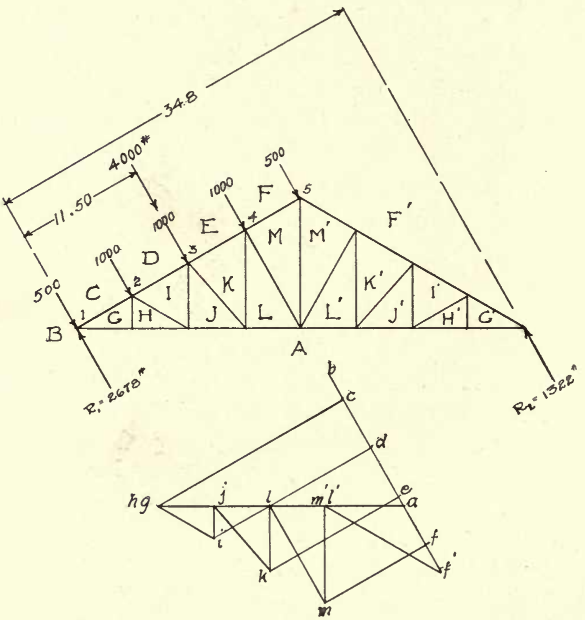


FIGURE 69

considered as acting at panel point No. 3. Its lever arm is 11.5 feet. The moment caused by it is 46,000 foot-pounds. The lever arm of R_2 around R_1 is 34.8 feet. It must be remembered that a moment is the product of a force multiplied by the *perpendicular* distance from the center of moments. R_1 is the center of moments in this case. R_2 multiplied by 34.8 must equal 46,000 foot-pounds. So $46{,}000 \div 34.8 = 1{,}322 = R_2$. $4{,}000 - 1{,}322 = 2{,}678$ pounds $= R_1$.

Once the reactions are determined, they can be laid off on the stress diagram. R_2 in this case is $F'A$ and $f'a$ laid off on the line bf' places a just below e. ab gives the left reaction, which equals 2,678

pounds. We now have *bc*, *cd*, *de*, *ef*, *ff'*, *f'a* and *ab*. The remainder of the stress diagram is laid off in the same manner as those given before. As the points *g* and *h* coincide with each other, the member *GH* in the truss diagram has no stress in it and might as well have been omitted, except that it acts as a support to keep the lower members of the truss from sagging. It will also be found that there will be no stress in *m'l'*, *l'k'*, and like members on the right-hand side of the truss when the wind is from the left. This is shown by the fact that *m'* and *l'* coincide with each other.

In order to make use of the diagrams already drawn it is necessary to make a table. In this table there must be spaces for the members of the truss and opposite each member is recorded the stress in it, first, when the load is vertical, second, when the wind blows from the right, third, when the wind blows from the left, and the sum of those stresses that give the greatest total stress. Each stress must be marked plus or minus to indicate if it is a tensile or compressive stress. The actual design of the members is then undertaken.

So far no attention has been paid to the condition in which one end of the truss is free. This problem will be dealt with in the next chapter as well as a few considerations dealing with the design of the truss.

CHAPTER XII

Wind Load Diagram — One end free. Tabulating of stresses. Cantilever Trusses.

IN order to establish the reactions in case one end of a truss is fixed and the other end free, the same method is employed as in the case of a simple beam, but, because the wind load is considered as acting in a direction perpendicular to the upper chord of the truss, and the right reaction, — R_2 — which is supporting the free end, acts upward in a vertical direction, there is a tendency to consider that this process is very difficult.

Fig. 70 is the same as Fig. 69 except the right end of the truss is considered as resting on rollers or on an iron plate over which it can

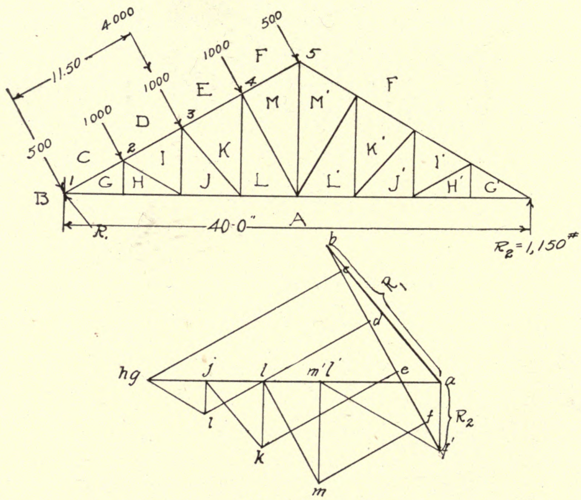


FIGURE 70

slide. The advantage of this is that allowance is made for expansion or contraction due to changes of temperature. If there is no restriction of the action of the stress in a horizontal direction, the only action that can be developed at R_2 is an upward one. It is therefore necessary to determine what this upward action is.

The moment around R_1 caused by the wind is exactly the same as in the case where both ends of the truss are fixed. The load is still

4,000 pounds and is considered as acting at panel point No. 3. The lever arm is 11.5 feet. The moment is therefore the same as before — 46,000 foot-pounds. To withstand this moment an opposite moment must be caused by R_2.

As R_2 in this case acts upwards, the *perpendicular* distance from R_1 to R_2 is 40 feet. The equation which gives the magnitude of R_2 is 46,000 foot-pounds $= R_2 \times 40$ feet, or, $R_2 = 46{,}000 \div 40 = 1{,}150$ pounds.

Because the condition is applied to a truss, and, because the forces are spoken of as wind loads and reactions, there is a tendency to make a symmetry of this example. If the same conditions were applied to a compound lever, such as found in automobiles, the

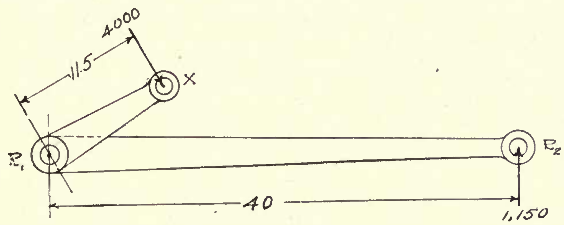


FIGURE 71

mystery may be explained in a more simple manner. In Fig. 71 such a lever is shown which is supposed to be pivoted at R_1, around which point it can swing freely and a load of 4,000 pounds is supposed to be applied at X in a direction perpendicular to R_1X. This force would tend to swing the lever around R_1, a moment of 46,000 foot-pounds having been created. But a force is exerted upward at R_2 which tends to hold the lever in its position. This force has a lever arm of 40 feet. How much force must be exerted at R_2? This is exactly the same example as given in the preceding case.

Once R_2 is determined the stress diagram is drawn *bc*, *cd*, *de*, *ef*, and *ff'* are laid off as in Fig. 69, but *f'a* offers a new problem. As has been said *F'A*, the right reaction, is a vertical force and so *f'a* must be drawn *upward*. The length of the line is determined by the magnitude of *F'A* or 1,150 pounds. The architect may have noticed that R_1 has not yet been determined, but all that it is necessary to do in order to establish both the direction and magnitude of the left reaction is to connect *a* and *b*. The line *ab* gives the neces-

sary information about *AB*, or R_1. Once all the external forces are plotted, the next step is the determination of the stresses in the members. This is done in exactly the same manner as in Fig. 69. It will be noticed that *cg*, *di*, *ck*, *fm*, etc., are of the same magnitude as in the case where both ends of the truss are fixed. The only members in which the stresses are different are those in the lower chord *GA*, *HA*, *JA*, and *LA*.

When both ends of the truss were fixed only one stress diagram was necessary. When the wind blows from the right the stresses

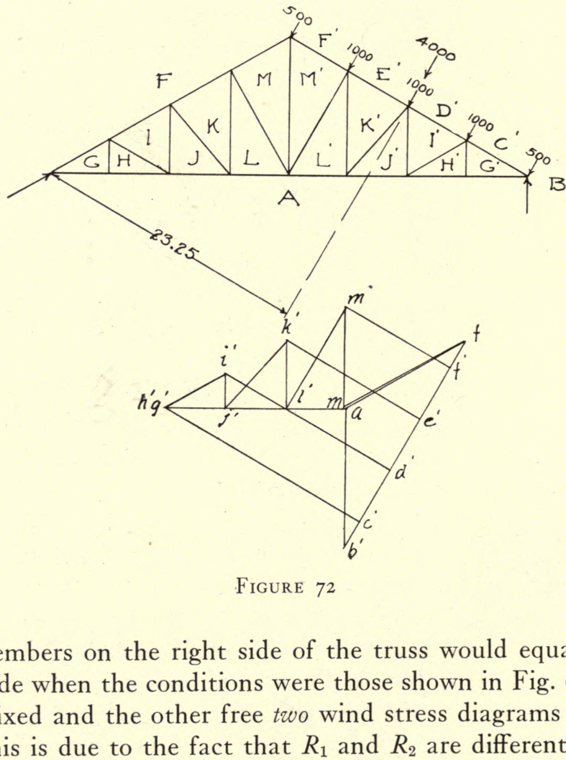


FIGURE 72

in the members on the right side of the truss would equal those in the left side when the conditions were those shown in Fig. 69. With one end fixed and the other free *two* wind stress diagrams are necessary. This is due to the fact that R_1 and R_2 are different when the wind blows from opposite sides of the roof.

The method of finding R_2 in the case where the wind blows from the right needs but little explanation. Fig. 72 is like that of the case where the wind is from the left, the only difference being that the lever arm of the wind load is longer. This follows from the fact that the direction of the wind load is different, and the *perpendicular*

distance from R_1 to the 4,000-pound load is 23.25 feet. The moment is 4,000 pounds × 23.25 feet = 93,000 foot-pounds.

The lever arm of R_2 is the same as before, 40 feet, so R_2 must equal 93,000 ÷ 40 = 2,325 pounds.

If it is desired to show how this condition is represented by a compound lever, note the diagram shown in Fig. 73. The lever arms and loads are represented in the most simple manner.

Once R_2 is determined the stress diagram is plotted — Fig. 72. It will be noticed that the loads start with FF' and read in the stress diagram in the following order: ff', $f'c'$, $c'd'$, $d'c'$, $c'b'$, $b'a$, and af.

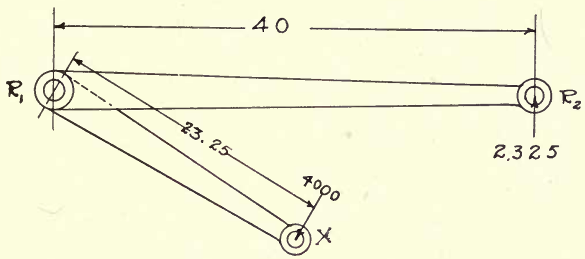


FIGURE 73

The left reaction — R_1 — is given by the last named line — af —and its direction is shown on the truss diagram.

In the two wind-load diagrams and the dead-load diagram shown in Fig. 74 all the stresses that may occur in the truss are shown. It now remains for the architect to scale the lines which show the stresses and to tabulate them as shown in Fig. 75. This table is self explanatory. The stresses are recorded in the *total* column and the signs denoting them represent the kind of stress in each member. These stresses are the *largest* that will occur. Each total is not the sum of all the stresses, but represents the sum of the dead-load stress plus the *greatest* wind-load stress.

To design the members of the truss it is necessary to know the *kind* of stress that occurs in each. If the member is in tension all that is necessary to do is to divide the stress by 16,000 pounds and the number of square inches necessary in each particular member is given.

On the other hand the compressive stress tend to buckle members in which they occur and these members must be designed as small columns. The formula $S = 15{,}200 - 58l/r$ is used in the same

manner as given in a previous chapter. For the purpose of simplifying the design of the truss the upper chord is made of angles that will withstand the stress occurring in *CG* although the stresses in *DI*, *EK* and *FM* are smaller than this. The same is true of the angles in the lower chord, the stress that governs the design is found in *GA*. Although smaller stresses may occur in members on the left side or

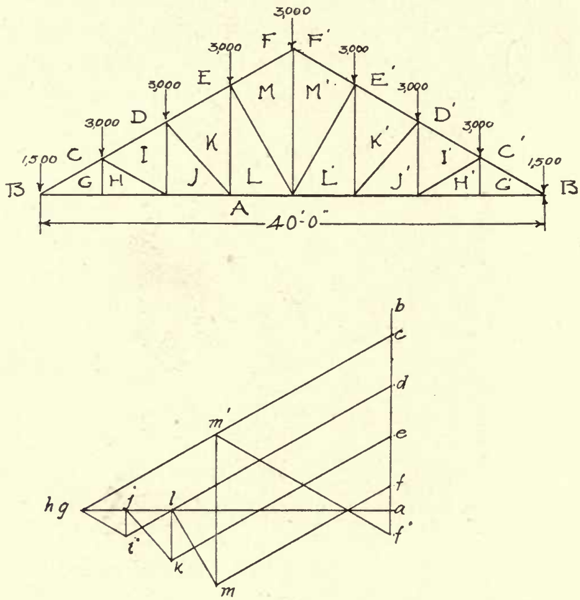


FIGURE 74

right side of the truss, as the case may be, both sides are made alike.

It is not necessary to design all the details of the truss as the steel contractor will furnish shop drawings of these. It is necessary, however, to give the stresses that occur and to know the number of rivets and size of members required.

It will be found in some tension members that the angles which will be strong enough to take up the stress will be too small to be riveted. In this case angles having legs of least 2½ inches must be considered the smallest that are practical.

Because of the need of only small members where tension exists, trusses are designed with tie rods to withstand the tensile stresses.

Such trusses are called "pin trusses." The compression members are made of deck beams or angles or of some other cast or rolled shapes.

Often in galleries in theaters there is need of cantilever trusses. These offer two new considerations for the architect to deal with. First there is the proposition of unsymmetrical loads and, second, the fact that either R_1 or R_2 may not act at the end, but somewhere near the center of the truss.

This last fact gives rise to the necessity of determining the reactions and the method used is that of finding the reactions of a simple cantilever beam. For the purpose of this chapter the truss shown in Fig. 76 will explain the method of solving such a problem. Loads of 1,000 pounds each are assumed as acting at the panel points 1–6,

MEMBER	DEAD LOAD	WIND LOAD L.	WIND LOAD R.	TOTALS
C G	+ 21,000	+ 3,600	+ 2,300	+24,600
D I	+ 18,000	+ 3,200	+ 2,300	+21,200
E K	+15,000	+ 2,600	+ 2,300	+17,600
F M	+12,000	+ 2,100	+ 2,300	+14,100
G A	−18,500	− 5,000	− 1,000	−13,500
H A	− 18,500	− 5,000	− 1,000	−13,500
J A	− 15,750	− 4,000	− 1,000	−19,750
L A	− 13,000	− 3,000	− 1,000	−16,000
H I	+ 3,000	+ 1,175	0	+ 1,175
I J	− 1,500	− 600	0	− 2,100
J K	+ 4,000	+ 1,500	0	+ 5,500
K L	− 3,000	− 1,200	0	− 4,200
L M	+ 5,000	+ 2,000	0	+ 7,000
M M'	− 9,000	− 1,800	− 1,700	−10,800

FIGURE 75

and loads of 500 pounds each acting at panel points 0 and 7. R_2 acts under panel point 4. Taking moments around panel point 0 the total downward moment is given as:

500 pounds × 0 feet = 000 foot-pounds.
1,000 " × 10 " = 10,000 "
1,000 " × 20 " = 20,000 "
1,000 " × 30 " = 30,000 "
1,000 " × 40 " = 40,000 "
1,000 " × 50 " = 50,000 "
1,000 " × 60 " = 60,000 "
500 " × 70 " = 35,000 "
7,000 pounds 245,000 foot-pounds.
245,000 foot-pounds ÷ 40 feet = 6,125 pounds.
7,000 − 6,125 = 875 "

It will be noticed that almost all the entire load comes upon R^2, and that had there been a greater load or a greater projection beyond R_2 there might have been a negative or downward reaction at R_1. The force *ab*, *bc*, *cd*, *de*, *ef*, *fg*, *gh*, and *ho* are laid off. The upward reaction — R_2 — *o'o* causes *o* to fall just below *a* and the left reaction

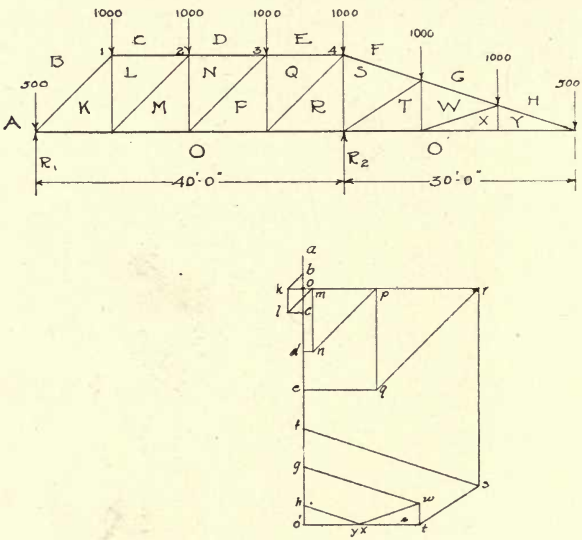


FIGURE 76

— R_1 — closes the force diagram. The stresses are laid off in the same manner as in the preceding diagrams.

To find the forces one may start at either end of the truss. Starting at the right end and reading this first the forces and stresses can be laid off as follows: *ho'*, *o'y* and *yh*. There will be no stress in *XY* and the stresses around panel point 6 can next be found. No difficulty will be experienced until R_2 is reached and there need be no trouble here if it is remembered that the force of 6,125 pounds — *o'o* — is read *upward.*

If there should be a case in which there is a negative moment at R_1 as is shown in Fig. 77 the reactions are determined as before. The three 1,000-pound loads and 500-pound load will tend to bend the truss around R_2 and to find the magnitude of this reaction take moments around R_1.

1,000 pounds	× 8 feet	=	8,000 foot-pounds.
1,000 "	× 14 "	=	14,000 "
1,000 "	× 20 "	=	20,000 "
700 "	× 26 "	=	13,000 "
3,500 pounds			55,000 foot-pounds.

$$55{,}000 \div 8 = 6{,}875 \text{ pounds.}$$

$$3{,}500 - 6{,}875 = -3{,}375 = R_1.$$

The fact that R_1 is a minus quantity shows that R_1 acts in an opposite direction to R_2. In laying off the forces it will be noticed

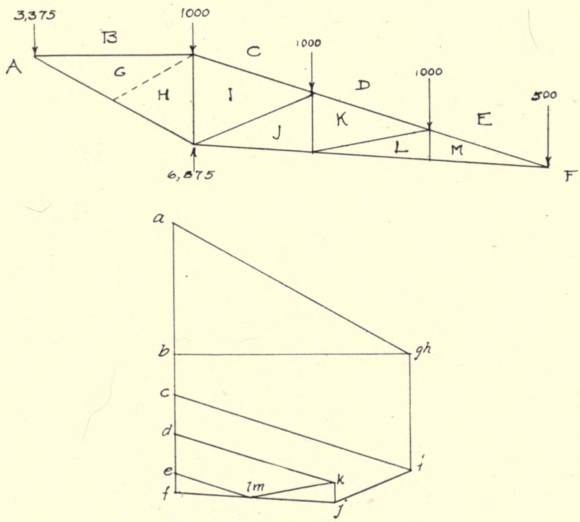


FIGURE 77

that there is only one upward force — R_2. Starting at the first joint *ab* is plotted. *bg* is a horizontal line and *ga* closes the diagram.

It will be found that there is no stress in *GH* as this is simply a bracing member. At 2 *gh* is known and *bc*. *ci* can be drawn through *c* and *ib* can be laid off.

Laying off the forces and stresses around the lower joint, *hi* is known. *ij* is drawn indefinitely through *i* in a direction parallel to *IJ*. The joint *f* is known on the stress diagram and so *jf* can be plotted. The intersection of these two lines gives the point *j*. *fa* is the upward reaction — R_2 — and *ah* has already been drawn.

Around panel point 3 *ji*, *ic* and *cd* are already established. *dk*

and *kj* are easily found. The stresses in other members can be found without any particular difficulty.

In this book the considerations taken up have been just those with which the architect comes in contact. There are cases in which the engineering requirements of an undertaking are such that special knowledge is necessary to solve the problems that come up. In this case it is always necessary for the architect to call upon an engineer to design the steel.

If, however, the architect has found in this book such information as will enable him, with the use of a steel handbook, to design the frames for simple architectural structures, the object for which the book was written has been accomplished.

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